FIXED ADJUSTMENT COSTS AND AGGREGATE FLUCTUATIONS

Michael W. L. Elsby  Ryan Michaels
University of Edinburgh  University of Rochester

December 23, 2014

Abstract

This paper studies the analytics of a canonical model of lumpy microeconomic adjustment. We provide a novel characterization of the implied aggregate dynamics. In general, the distribution of firm outcomes follows a simple and intuitive law of motion that links aggregate outcomes to rates of adjustment. Analytical approximations reveal, however, that the aggregate dynamics are approximately invariant to a relevant range of adjustment costs. This neutrality is an aggregation result that emerges from a symmetry property in the distributional dynamics, independent of market equilibrium considerations. Quantitative illustrations confirm these results for parameterizations used in the employment and price adjustment literatures.

JEL codes: E24, E3, J23, J63.

Keywords: Lumpy microeconomic adjustment, S-s model, cross-sectional dynamics, aggregate employment dynamics.

*We thank Rudi Bachmann, Giuseppe Bertola, Russell Cooper, William Hawkins, Virgiliu Midrigan, and David Ratner for very helpful comments, and seminar participants at the Federal Reserve Bank of New York, the May 2011 Essex Economics and Music conference, the Bundesbank/European Central Bank/Centre for Financial Studies joint seminar, Western Ontario, Royal Holloway, the European University Institute, Southampton, Exeter, III Workshop on “Industry and Labor Market Dynamics” Barcelona March 2012, Maryland, University of Texas at Austin, Carlos III, the 2012 Society for Economic Dynamics Meetings, Duke, Yeshiva, UCL, LSE, Koc, NYU Brown Bag, Manchester, the 2014 Royal Economic Society conference, and Penn State. All errors are our own. E-mail addresses for correspondence: mike.elsby@ed.ac.uk, and ryan.michaels@rochester.edu.
Inaction in microeconomic adjustment is pervasive. A stylized fact of the empirical dynamics of employment, investment and prices is that they exhibit periods of inaction punctured by bursts of adjustment.\footnote{A non-exhaustive summary includes: Hamermesh (1989), Caballero, Engel and Haltiwanger (1997), Cooper, Haltiwanger and Willis (2005, 2007), and Bloom (2009) on employment; Doms and Dunne (1998), Caballero, Engel and Haltiwanger (1995), Cooper and Haltiwanger (2006), and Bloom (2009) on capital; Bertola and Caballero (1990) and Bertola, Guiso and Pistaferri (2005) on durable goods; and Bils and Klenow (2004) and Nakamura and Steinsson (2008) on prices.} A leading explanation of this phenomenon is that firms face a fixed cost of adjusting.\footnote{Of course, fixed adjustment costs are not the only explanation of the observed inaction in microeconomic adjustment. An alternative possibility is that adjustment involves discrete marginal costs, such as in the kinked adjustment cost case. See, for example, Bertola and Caballero (1994b).} In such an environment, firms will choose not to adjust for some time, with periodic discrete adjustments in response to sufficiently large shocks, consistent with the empirical “lumpiness” of microeconomic dynamics.

In this paper, we analyze the aggregate implications of this lumpiness at the microeconomic level. We do so in the context of a canonical model of fixed adjustment costs in the presence of aggregate and idiosyncratic shocks that has been used widely in prior literature. For concreteness, we focus on the case of employment adjustment, although we show how the model can be applied equally to prices and investment dynamics.

We establish a novel neutrality result: Even in the absence of equilibrium adjustment of market prices, both the steady-state and dynamic aggregate outcomes implied by standard models are approximately neutral with respect to a plausibly small fixed adjustment cost.

The remainder of the paper proceeds as follows. In section 1, we describe the basic ingredients of the model. Firms face shocks to labor productivity that induce changes in their desired level of employment. Firms are subject to both aggregate and idiosyncratic shocks. Aggregate shocks drive macroeconomic expansions and recessions; idiosyncratic shocks drive heterogeneity in employment dynamics across firms. Due to the presence of a fixed adjustment cost, however, firms’ employment will not adjust in response to all shocks. Instead, employment evolves according to an Ss policy at the microeconomic level, remaining constant for intervals of time with occasional jumps to a new level.

Given this environment, Section 2 develops the first result of the paper. Specifically, it takes on the task of aggregating the lumpy microeconomic behavior identified in section 1 up to the macroeconomic level. These aggregate implications are not obvious. Since individual firms follow highly nonlinear Ss labor demand policies, and face heterogeneous idiosyncratic productivities, there is no representative firm interpretation of the model.

We show how it is possible to infer the dynamics of aggregate employment by solving for the dynamics of a related object, namely the cross-sectional distribution of employment...
across firms. By applying a simple mass-balance approach, we provide a transparent characterization of the distribution dynamics of employment. Two aspects of the result are novel. First, it holds for a comparatively wide class of processes for shocks and adjustment rules.\(^3\) Second, and perhaps more importantly, our characterization of aggregate dynamics admits a particularly clean economic interpretation. In particular, we show that the evolution of the firm-size distribution can be related simply and intuitively to the probabilities of adjusting to and from each employment level. By impeding these flow probabilities, the adjustment friction distorts the firm-size distribution. These dynamics of the distribution of employment across firms in turn shape the evolution of aggregate employment, since the latter is simply the mean of that distribution.\(^4\)

This characterization of the cross section greatly facilitates our subsequent analysis of the model’s dynamics. In Section 3, we apply our characterization to develop the second main result of the paper—approximate aggregate neutrality. In particular, we use the general results of section 2 to inform analytical approximations to model outcomes in the presence of a small fixed adjustment cost. This is a compelling neighborhood to study because, as noted since Akerlof and Yellen (1985) and Mankiw (1985), even small adjustment costs will induce substantial inaction in microeconomic adjustment. In this neighborhood, we show that both the steady-state and the dynamics of the firm-size distribution approximately coincide with their frictionless counterparts. It follows that the same approximate neutrality extends to the behavior of aggregate employment in general.

We show that this approximate neutrality result can be traced to a symmetry property that emerges in the distributional dynamics of employment as the adjustment friction becomes small. The mass-balance approach of section 2 makes the intuition for this symmetry particularly transparent. Specifically, the change over time in the mass of firms at a given level of employment can be decomposed into an inflow of firms that adjusts to that level, less an outflow of firms that adjust away from that level of employment. The key is that a fixed adjustment cost reduces both of these flows relative to the frictionless case. Fewer firms adjust away from a given employment level. But, in addition, fewer firms find it optimal to adjust to that employment level. For small frictions, these two forces are symmetric, leaving the distribution of employment approximately equal to its frictionless counterpart.

---

\(^3\)Prior literature has studied aggregation for cases in which shocks are governed by specific processes (e.g. Brownian motion; Bertola and Caballero, 1990) or particular adjustment rules (e.g. the one-sided Ss case; Caballero and Engel, 1991). The latter papers analyze environments that are more general in other dimensions, however. Bertola and Caballero (1990) study the case with both fixed and kinked adjustment costs; Caballero and Engel (1991) allow for heterogeneity in individual policy rules.

\(^4\)As we discuss in section 2, our approach is related to the Kolmogorov equations in continuous-time models. The advantage of our approach is that it reveals a clear economic interpretation of the dynamics.
This neutrality result is reminiscent of Caplin and Spulber (1987) who obtain a similar outcome in a related pricing problem. Although they consider a much simpler environment without idiosyncratic shocks and only one-sided adjustment, our result retains a flavor of theirs. Specifically, Caplin and Spulber demonstrate that a uniform cross-sectional distribution will be invariant in their model due to a form of symmetry—firms induced to adjust from the bottom of the distribution to the top exactly replace firms displaced from the top of the distribution. Thus, one interpretation of our neutrality result is that it generalizes the Caplin and Spulber insight to an environment with idiosyncratic risk and two-sided adjustment. By the same token, this helps to explain why Golosov and Lucas (2007) find small aggregate effects in their quantitative analysis of a related model with these ingredients.

An interesting feature of our approximate aggregate invariance result is that it holds for any realization of the aggregate state of the economy, which includes firms’ perceptions of the current and future path of the equilibrium wage. That is, it does not rely on equilibrium adjustment of wages. This contrasts with an influential recent literature that has emphasized the role of market price adjustment in muting the aggregate effects of fixed adjustment costs (see, for example, Khan and Thomas, 2008; Veracierto, 2002; House, 2008). Rather, the near-symmetry in the equilibration of the distribution of firm size is a property of aggregation, and thereby holds for any configuration of market prices.

In section 4 of the paper, we illustrate these analytical results in a series of quantitative illustrations. We first parameterize the model using estimates from recent literature on employment adjustment and firm productivity (Bloom, 2009; Cooper, Haltiwanger and Willis, 2005, 2007; Foster, Haltiwanger and Syverson, 2008). Numerical results reveal that this parameterization of the model implies aggregate employment dynamics that are very close to their frictionless analogue even when market wages are fixed, in line with the approximate-neutrality result in section 3.

There remains a lack of consensus over some of the parameters of the model, however, so we also explore the sensitivity of this baseline result. Alternative parameterizations that match the frequency and average size of employment adjustments in U.S. microdata; vary the persistence of idiosyncratic shocks; and allow for different specifications in which adjustment costs vary stochastically over time, or with firm size, all leave the approximate neutrality result in baseline case essentially unimpaired.

To generate deviations from frictionless dynamics, the model suggests that rates of ad-

---

5In the case of a price setting problem, the aggregate state incorporates firms’ anticipations of future aggregate prices. For any set of these anticipations, our neutrality result implies that the aggregate supply curve will approximately coincide with its frictionless counterpart.
justment must be significantly lower. Consistent with this, we find that the dynamics of aggregate employment can exhibit some persistence relative to its frictionless counterpart in the case where the adjustment cost is larger relative to the variance of innovations to idiosyncratic productivity. This mirrors Alvarez and Lippi’s (2014) emphasis in related research on the importance of idiosyncratic dispersion to aggregate dynamics. We find, though, that the effects of alternative, plausible parameterizations are modest, yielding only small deviations from frictionless dynamics that vanish after a quarter or two. Taken together, these results suggest that the symmetry result uncovered in section 3 is quite powerful, in the sense that it is robust to a number of alternative parameterizations.

In the closing sections of the paper, we show how our analytical framework can be used to elucidate cases in which non-neutralities emerge. A few recent papers have considered a Poisson-like process for idiosyncratic productivity in which firms draw a new value with some probability each period (Gertler and Leahy, 2008; Midrigan, 2011). This induces an atom in the conditional distribution of idiosyncratic productivity at its lagged value. We show that this discontinuity in turn breaks neutrality in a precise way. Specifically, we show that the symmetry, and hence also the neutrality we emphasize in the early sections of the paper, hold for all firms except those prevented from adjusting by the Poisson friction. It follows that aggregate employment evolves approximately according to a pure partial-adjustment process with constant rate of convergence equal to the Poisson parameter, mirroring Calvo dynamics. A quantitative illustration confirms the accuracy of this prediction.

We conclude by highlighting promising avenues of future research in the light of our findings. One message is that the role of the magnitude of adjustment frictions relative to idiosyncratic uncertainty in shaping implied aggregate dynamics emphasizes the value of obtaining robust estimates of these parameters. Beyond this, though, the unifying theme of symmetry that underlies the results of this paper provides two further directions to pursue. First, more work that assesses the presence of asymmetries in firms’ adjustment policies and their contribution to deviations from frictionless dynamics would be worthwhile. Second, further empirical research into the distributional form of idiosyncratic shocks also will shed an important light on the aggregate consequences of fixed adjustment costs.

---

6This dovetails with prior numerical results. See, for example, King and Thomas (2006) and Gourio and Kashyap (2007), who consider models that abstract from idiosyncratic heterogeneity in productivity, and Bachmann (2013), who considers a model with a smaller adjustment rate.

7In Appendix B we also report results for the case with equilibrium price adjustment. This confirms the message of King and Thomas (2006) and Khan and Thomas (2008) that, where non-neutralities exist for fixed market prices, equilibrium price adjustment pushes the dynamics toward their frictionless path.

8A particularly interesting possibility is that the asymmetry in the adjustment hazards estimated by Caballero, Engel and Haltiwanger (1995, 1997) plays a role in aggregate outcomes.
1 The Firm’s Problem

We consider a canonical model of fixed employment adjustment costs. Later, we describe how our analysis can be applied to related problems of capital and price adjustment. The microeconomic environment is as follows. Time is discrete. Firms use labor, \( n \), to produce output according to the production function, \( y = pxF(n) \), where \( p \) represents the state of aggregate labor demand, \( x \) represents shocks that are idiosyncratic to an individual firm, and the function \( F \) is increasing and concave, \( F_n > 0 \) and \( F_{nn} < 0 \). We assume the evolution of idiosyncratic shocks is described by the density function \( g(x'|x) \) with associated distribution function \( G(x'|x) \).

At the beginning of a period, firms observe the realization of their idiosyncratic shocks \( x \), as well as aggregate productivity \( p \). Given this, they then make their employment decision. If the firm chooses to adjust the size of its workforce, it incurs a fixed adjustment cost, denoted \( C \).

For the purposes of the main text, we focus on the case in which there is no exogenous attrition of a firm’s workforce, so that during periods of inaction employment remains unchanged. We do this to economize on notation and to convey ideas transparently. The Appendix shows that all the results we present continue to hold for the case with attrition.

It follows that we can characterize the expected present discounted value of a firm’s profits recursively as:

\[
\Pi(n_{-1}, x; \Omega) \equiv \max_n \left\{ pxF(n) - wn - C1 + \beta \mathbb{E}[\Pi(n', x'; \Omega') | x; \Omega] \right\}, \tag{1}
\]

where \( 1 \equiv 1 \) \( [n \neq n_{-1}] \) is an indicator that equals one if the firm adjusts and zero otherwise. The wage \( w \) is determined in a competitive labor market, and is taken as exogenous from the firm’s perspective. The variable \( \Omega \) summarizes the aggregate state of the economy. The latter includes the wage \( w \), the aggregate shock \( p \), and all variables that are informative with respect to their future evolution, including, for instance, the preceding periods’ firm size distributions.

For the analysis that follows, it is helpful to recast the firm’s problem in equation (1) into two related underlying Bellman equations. In particular, the value of adjusting (gross

\footnote{It is common in the literature to scale the adjustment cost by some measure of productivity. Coupled with other assumptions on the stochastic process of \( x \), this scaling makes the problem homogeneous in \( x \), thereby allowing one to eliminate a state variable (see, for example, Caballero and Engel, 1999, and Gertler and Leahy, 2008). We focus on a pure lump-sum adjustment cost \( i \) to simplify the analysis; and \( ii \) to highlight that our results do not rely on homogeneity of the value function. In section 4, we discuss how the insights derived from the lump-sum case carry over to a scale-dependent cost.}
of the adjustment cost), $\Pi^\Delta (x; \Omega)$, and the value of not adjusting, $\Pi^0 (n_{-1}, x; \Omega)$, are given by

$$
\Pi^\Delta (x; \Omega) \equiv \max_n \{ pxF (n) - wn + \beta \mathbb{E} [\Pi (n, x'; \Omega') | x, \Omega] \}, \text{ and}
$$

$$
\Pi^0 (n_{-1}, x; \Omega) \equiv pxF (n_{-1}) - wn_{-1} + \beta \mathbb{E} [\Pi (n_{-1}, x'; \Omega') | x, \Omega].
$$

Clearly, the value of the firm $\Pi (n_{-1}, x; \Omega)$ is simply the upper envelope of these two regimes,

$$
\Pi (n_{-1}, x; \Omega) = \max \{ \Pi^\Delta (x; \Omega) - C, \Pi^0 (n_{-1}, x; \Omega) \}.
$$

In keeping with the literature on fixed adjustment costs, we assume that the optimal labor demand policy takes an $S$s form. Figure 1 illustrates such a policy. It is characterized by three smooth functions, $L(n; \Omega) < X(n; \Omega) < U(n; \Omega)$. In the event that the firm chooses to adjust away from $n_{-1}$, optimal employment is determined by a “reset” function $X(n; \Omega)$ which satisfies the first-order condition

$$
pX (n; \Omega) F_n (n) - w + \beta \mathbb{E} [\Pi_n (n, x', \Omega') | x = X (n), \Omega] \equiv 0.
$$

It follows that the labor demand schedule, conditional on adjusting, is $X^{-1} (x; \Omega)$. In other words, the reset function is the (conditional) inverse labor demand schedule.

Due to the adjustment cost, however, the firm will not always choose to adjust: It will decide to adjust only if the value of adjusting, net of the adjustment cost, $\Pi^\Delta (x; \Omega) - C$, exceeds the value of not adjusting, $\Pi^0 (n_{-1}, x; \Omega)$. This aspect of the firm’s decision rule is characterized by two adjustment “triggers,” $L(n_{-1}; \Omega)$ and $U(n_{-1}; \Omega)$. For sufficiently bad realizations of the idiosyncratic shock, $x < L(n_{-1}; \Omega)$, the firm will shed workers; for sufficiently good shocks, $x > U(n_{-1}; \Omega)$, it will hire workers. For intermediate values of $x \in [L(n_{-1}; \Omega), U(n_{-1}; \Omega)]$, the firm will neither hire nor fire, and $n = n_{-1}$. Thus, the adjustment triggers trace out the locus of points for which the firm is indifferent between adjusting and not adjusting. It follows that the triggers satisfy the value-matching conditions

$$
\Pi^\Delta (L(n_{-1}; \Omega); \Omega) - C = \Pi^0 (n_{-1}, L(n_{-1}; \Omega); \Omega), \text{ and}
$$

$$
\Pi^\Delta (U(n_{-1}; \Omega); \Omega) - C = \Pi^0 (n_{-1}, U(n_{-1}; \Omega); \Omega).
$$

---

$^{10}$It is difficult to prove the optimality of the $S$s policy in general. Exceptions are the continuous-time Brownian case (Harrison, Sellke and Taylor, 1983), and the case of one-sided adjustment (Scarf, 1959; Roys, 2014). However, we show later (Lemma 2) that the optimal policy is well-approximated by its myopic ($\beta = 0$) counterpart, which does take the $S$s form.

$^{11}$Clausen and Strub (2014) establish differentiability in $n$ for this problem.
Firms’ optimal policies clearly depend on the aggregate state $\Omega$. For example, positive shocks to $p$ will cause the $S_s$ policy in Figure 1 to shift downward: For any given $x$, a firm will be less likely to fire, and more likely to hire in an aggregate expansion.

## 2 Aggregation

This section infers the aggregate implications of firms’ $S_s$ labor demand policies. Aggregation in this context is non-trivial: an individual firm’s labor demand depends in a highly nonlinear fashion on its individual lagged employment $n_{-1}$ and the idiosyncratic shock $x$. Heterogeneity in these state variables implies there is no representative firm interpretation of the model.

To infer aggregate labor demand, we characterize a related object—the cross-sectional distribution of employment across firms. We denote the density of this distribution by $h(n)$, and its associated distribution function by $H(n)$. The aggregation result we develop in this section is an important ingredient to our subsequent analysis of the conditions under which aggregate outcomes are approximately neutral to the adjustment friction, $C$.

Our approach can be conveyed most transparently in the special case where $x$ is i.i.d., with smooth distribution function $G(x)$. To begin, we calculate the outflow of mass from the density, $h(n)$.

$$\int \frac{1 - G(U(n)) + G(L(n))}{h(n)} \, dn = \int (1 - G(U(n)) + G(L(n))) \cdot h_{-1}(n) \, dn.$$  

To infer the inflow of mass to $h(n)$, consider the set of firms that draw an idiosyncratic productivity of $x = X(n)$. If the adjustment cost were suspended momentarily, these firms would adjust to $n$, and the inflow of mass into $h(n)$ would equal $\partial G[X(n)] / \partial n \equiv h^*(n)$. Following Caballero, Engel and Haltiwanger (1995), we refer to $h^*(n)$ as the density of mandated employment.

In the presence of a fixed cost, however, Figure 1 reveals that only firms whose initial employment, $n_{-1}$, is either relatively low ($n_{-1} < U^{-1}X(n) < n$) or
relatively high \((n_{-1} > L^{-1}X(n) > n)\) will adjust to \(n\). Thus, the inflow of mass into \(h(n)\) is

\[
(1 - H_{-1} [L^{-1}X(n)] + H_{-1} [U^{-1}X(n)]) \cdot h^*(n),
\]

where \(H_{-1}(\cdot)\) denotes the distribution function of inherited employment. The change in the mass at \(n\), \(\Delta h(n)\), is then the difference between the inflows (8) and the outflows (7).

Proposition 1 generalizes this approach to the case in which idiosyncratic productivity \(x\) follows a first-order Markov process, with distribution function \(G(x'|x)\).

Proposition 1 (Aggregation) The density of employment across firms evolves according to the difference equation

\[
\Delta h(n) = (1 - \mathcal{H} [L^{-1}X(n) | X(n)] + \mathcal{H} [U^{-1}X(n) | X(n)]) \cdot h^*(n)
- (1 - \mathcal{G} [U(n) | n] + \mathcal{G} [L(n) | n]) \cdot h_{-1}(n),
\]

where \(\mathcal{G}(\xi|\nu) \equiv \Pr [x \leq \xi | n_{-1} = \nu]\) is the distribution function of idiosyncratic productivity conditional on start-of-period employment; \(\mathcal{H}(\nu|\xi) \equiv \Pr [n_{-1} \leq \nu | x = \xi]\) is the distribution function of start-of-period employment conditional on current idiosyncratic productivity; and \(h^*(n) \equiv \partial G[X(n)]/\partial n\) is the density of mandated employment.

Proposition 1 closely resembles the results from the i.i.d. case, except that the probabilities of adjusting to and from \(n\) are modified to account for persistence in \(x\). Initial firm size conveys information about past productivity through last period’s optimal employment policy. Since productivity is persistent, the probability of events, \(x \geq U(n)\) or \(x \leq L(n)\), must then be calculated conditional on initial size, \(n\). It follows that the probability of adjusting away from \(n\) is \(1 - \mathcal{G} [U(n) | n] + \mathcal{G} [L(n) | n]\), with \(\mathcal{G}\) defined as in Proposition 1. The outflows from \(n\) now take the form in (7), but with \(G\) replaced by \(\mathcal{G}\). In the same vein, the realization of \(x = X(n)\) conveys information about the distribution of lagged employment. Consequently, the probability of adjusting to \(n\) is evaluated according to the distribution, \(\mathcal{H}\), of lagged employment conditional on \(x = X(n)\). This yields \(1 - \mathcal{H} [L^{-1}X(n) | X(n)] + \mathcal{H} [U^{-1}X(n) | X(n)]\).\(^{14}\) The inflow of firms to \(n\) takes the form in (8) but with \(H_{-1}\) replaced by \(\mathcal{H}\).

Proposition 1 provides a link from the microeconomic friction to the aggregate dynamics. The fixed cost slows the movement of firms away from their initial size \(n\), since a share of

\(^{14}\)Of course, the distributions \(\mathcal{G}\) and \(\mathcal{H}\) are linked by Bayes’ rule. Lemma 3 in the Appendix establishes a law of motion for \(\mathcal{G}\) that preserves analyticity
them, $G[U(n)|n] - G[L(n)|n]$, does not find it profitable to adjust. Likewise, only a fraction of firms that desire to adjust to $n$ relocate there in the face of the fixed cost.

### 2.1 Aggregate labor demand

With the aid of Proposition 1, it is straightforward to construct aggregate labor demand. Based on the aggregate state $\Omega$, firms derive their optimal labor demand policy functions $L(n; \Omega) < X(n; \Omega) < U(n; \Omega)$. The aggregate implications of firm’s choices are expressed through the density of employment $h(n)$, computed as in Proposition 1, for a given history $h_{-1}(n)$. Aggregating over firms thus yields aggregate labor demand for a given aggregate state,

$$N^d(\Omega) = \int nh(n; \Omega) \, dn. \quad (10)$$

Proposition 1 delivers a key ingredient to labor market equilibrium. It is only one ingredient, however. Recall that the aggregate state $\Omega$ includes aggregate productivity $p$, the market wage $w$, and all variables that are informative with respect to their future evolution. Equilibrium requires two additional conditions that bear on $\Omega$, on which Proposition 1 is silent. First, the market wage $w$ adjusts, and is anticipated to adjust, to equate aggregate labor demand in (10) with aggregate labor supply at all points in time. Second, and related, firms’ perceptions of the aggregate state $\Omega$ must be consistent with equilibrium outcomes. In particular, since $\Omega$ includes any information that forecasts future wages, it follows from (10) that firms’ perceptions of the current (and expectations of the future) firm-size distribution $h(n)$ are part of the aggregate state. In equilibrium, these perceptions must in turn coincide with the law of motion reported in Proposition 1, evaluated at the equilibrium wage.\footnote{In practice, these fixed-point problems are difficult to solve, because the distribution is an infinite-dimensional object. To overcome this, quantitative implementations of such models often assume that firms are boundedly rational in the sense that they can form a forecast only of the mean, and use this to predict future wages (Krusell and Smith, 1998).}

Nonetheless, we shall see in section 3 that Proposition 1 sheds light on the aggregate equilibrium by uncovering properties of aggregate labor demand that hold for any $\Omega$. In particular, we establish that aggregate labor demand is approximately invariant to small fixed adjustment costs, in the sense that the aggregate labor demand schedule in equation (10) (approximately) coincides with its frictionless counterpart, for any set of perceptions about the current (and future evolution) of the aggregate state. It follows that the intersection of aggregate labor demand and supply will yield (approximately) the frictionless equilibrium.
2.2 Relation to the literature

We are not the first to consider the analytics of aggregating lumpy microeconomic behavior.\footnote{A much larger literature has instead used numerical methods to infer aggregate quantities. See, for example, Bachmann (2013), Golosov and Lucas (2007), Khan and Thomas (2008), among others.} For example, a number of papers have considered the implications of one-sided Ss policies in which the variable under control—employment in the above model—is adjusted only in one direction. As Cooper, Haltiwanger and Power (1999) and King and Thomas (2006) show, one-sided adjustment yields much simpler cross-sectional dynamics: Employment (or capital) at each firm decays exogenously, and is intermittently updated to a reset value. However, two-sided adjustment is a perennial feature of employment and price adjustment—firms hire and fire workers (Davis and Haltiwanger, 1992); prices are adjusted both up and down (Klenow and Malin, 2011). Proposition 1 provides a means to analyze the aggregate effects of adjustment frictions in this empirically-relevant case. We shall see that the presence of two-sided adjustment has important implications for the nature of aggregate dynamics.

In two-sided adjustment problems, progress on aggregation has been made within the class of continuous-time models where idiosyncratic shocks follow a Brownian motion.\footnote{In the special case in which shocks evolve according to Brownian motion, aggregation in models of lumpy adjustment can be derived using the Kolmogorov equations (see, for example, Dixit, 1993, and Dixit and Pindyck, 1994). An example of the application of these methods for the case of irreversible investment can be found in Bertola and Caballero (1994a).} Most recently, Alvarez and Lippi (2014) study a price-setting problem with multiple products and show that the dynamics of the average price level—alogous to the mean of \( h(n) \) in our context—are mediated by the frequency of adjusting, a result reminiscent of Proposition 1 above. Bertola and Caballero (1990) study aggregate outcomes in a Brownian model in which there are both fixed and kinked costs of adjusting (the latter are omitted in our analysis). Proposition 1 is not restricted to the Brownian class; rather, our results obtain for a general first-order Markov process for idiosyncratic productivity.

For our purposes, Proposition 1 is especially useful because it facilitates analysis of the aggregate dynamics in the next section. The simple link between the dynamics of the cross section and the adjustment probabilities to and from points in the distribution appears to be new to the literature, and provides a mapping from the microeconomic friction to the aggregate dynamics with a clean economic interpretation. We show how to use this result to characterize the model’s aggregate implications in a transparent way.
3 Approximate Aggregate Neutrality

The previous section provided a general characterization of the aggregate dynamics implied by a model of lumpy microeconomic adjustment. In this section, we derive analytical approximations to model outcomes that form the basis of the second key result of the paper, namely that the aggregate dynamics characterized in Proposition 1 are approximately neutral with respect to (that is, invariant to) the fixed adjustment cost.

3.1 Some preliminary lemmas

Our analysis in this section begins by describing two intermediate results that inform the neutrality result. These reveal two key properties of the firm's optimal labor demand policy in the neighborhood of a small fixed adjustment cost. The first reiterates the insights of Akerlof and Yellen (1985) and Mankiw (1985) to argue that the case of a small fixed cost is particularly instructive, because even small adjustment frictions imply substantial inaction, and hence lumpiness, in microeconomic adjustment.\textsuperscript{18}

Lemma 1 (Akerlof and Yellen, 1985) If the fixed adjustment cost $C$ is small—that is, orders greater than $C$ are negligible—the adjustment triggers and their inverses are approximately equal to

\begin{align*}
L (n) &\approx X (n) - \gamma (n) \sqrt{C}, \quad U (n) \approx X (n) + \gamma (\bar{n}) \sqrt{C}, \text{ and} \\
L^{-1} (x) &\approx X^{-1} (x) + \bar{\gamma} (x) \sqrt{C}, \quad U^{-1} (x) \approx X^{-1} (x) - \bar{\gamma} (x) \sqrt{C},
\end{align*}

\textit{where $\gamma (n) > 0$, $\bar{\gamma} (x) > 0$, and $\gamma (n) = X' (n) \bar{\gamma} (X (n))$.}

Lemma 1 implies that the adjustment triggers and their inverses that feature prominently in Proposition 1 display an approximate \textit{symmetry} in the square root of the adjustment friction. It follows that even second-order small adjustment costs—that is, $C = \varepsilon^2$—generate first-order inaction bands—for example, $U (n) - X (n) \propto \varepsilon$. The functions $\gamma (n)$ and $\bar{\gamma} (x)$ reflect the curvature in the return to adjusting, and therefore mediate the effect of the adjustment cost on the adjustment triggers. They are linked by the change of variables relation $\gamma (n) = X' (n) \bar{\gamma} [X (n)]$, which maps units of employment to units of productivity.

\textsuperscript{18}Lemma 1 requires that the values of adjusting and inaction in (6) are twice-differentiable in $n$ and $x$. This is implied by our assumption, in keeping with literature, that the adjustment triggers $L (n)$ and $U (n)$ are smooth functions of $n$. Lemma 2 will further show that the optimal adjustment triggers are approximated by the static ($\beta = 0$) policy, which satisfies this smoothness requirement.
The second intermediate result we will exploit extends the original insights of Gertler and Leahy (2008) to provide a sharper characterization of the optimal policy. A corollary of Gertler and Leahy’s Simplification Theorem for our environment is that the optimal policy approximately coincides with its myopic (that is, $\beta = 0$) counterpart in the neighborhood of a small fixed adjustment cost. That is, an excellent approximation to optimal dynamic labor demand can be obtained simply by solving for the functions $L(n)$, $X(n)$, and $U(n)$ associated with the corresponding static problem. As stressed by Gertler and Leahy, an important ingredient in this result is the presence of two-sided adjustment—that is, that both upward and downward adjustments occur with positive probability in each state. Gertler and Leahy’s analysis, however, restricts attention to a particular process of idiosyncratic productivity shocks $x$. Lemma 2 establishes the approximate optimality of myopia for any first-order Markov process.

**Lemma 2 (Gertler and Leahy, 2008)** In the presence of two-sided adjustment, the future value of the firm is independent of current employment, $n$, up to terms greater than order $C$.

The intuition behind the result is straightforward. Note first that current employment affects future profits only in the event that the firm does not adjust in the subsequent period—that is, if $x' \in [L(n), U(n)]$. From Lemma 1, the width of the inaction band is of order $\sqrt{C}$. One can show, then, that the probability of inaction is also of order $\sqrt{C}$. In addition, by optimality, the return to inaction realized in this event, $\Pi^0(n, x') - [\Pi^A(x') - C]$, is of order $C$—it must be bounded from below by zero (otherwise the firm will choose to adjust) and from above by the adjustment cost $C$ (since inaction cannot dominate costless adjustment, $\Pi^0(n, x') \leq \Pi^A(x')$). It follows that the effect of $n$ on the future value of the firm, via its role in the expected value of inaction, is of order $C^{3/2}$. The effect of ignoring this term on the firm’s profits is negligible, and the firm’s problem thus approximates the $\beta = 0$ case.

The key implication of Lemma 2 for what follows is that the reset policy $X(n)$ approximately coincides with its frictionless counterpart, since it satisfies the frictionless first-order condition, $pX(n) F_n(n) \equiv w$. Thus, $h^*(n)$ may now be interpreted as the frictionless density of employment that would result if the adjustment cost were suspended indefinitely, and not just the distribution mandated by the reset policy if the adjustment cost were suspended momentarily. For this reason, henceforth we will refer to $h^*(n)$ as the frictionless density.

---

19Gertler and Leahy assume that shocks to $x$ arrive each period with a given probability and, conditional on arrival, follow a geometric random walk with uniform innovations. The approximate optimality of myopia emerges when the probability of arrival equals one. In our case, shocks to $x$ arrive every period, but evolve according to a general first-order Markov process. In section 4, we return to consider a compound-Poisson process of the type assumed by Gertler and Leahy and Midrigan (2011).
3.2 The neutrality result

We are now prepared to state the main result of this section, and the second key result of the paper, which demonstrates that aggregate dynamics are approximately neutral to the adjustment cost. To derive this result, we assume that the distribution of idiosyncratic shocks \( g(x'|x) \), as well as the the initial density of firm size \( h_{-1}(n) \), are analytic functions—that is, that they can be represented by Taylor series expansions. This facilitates the approximations required to establish the result. This assumption is consistent with conventional parameterizations used in the literature, which typically invokes lognormal shocks. Later, in section 4, we examine the implications of violations of analyticity for a compound-Poisson process for \( x \) proposed in recent literature (Gertler and Leahy, 2008; Midrigan, 2011).

Proposition 2 (Neutrality) Assume \( g(x'|x) \) and \( h_{-1}(n) \) are analytic functions, and that adjustment is two-sided. Then, for any aggregate state \( \Omega \), a first-order approximation around \( C = 0 \) to the evolution of the distribution of employment across firms is given by

\[
\Delta h(n) \approx -[h_{-1}(n) - h^*(n)],
\]

which is the frictionless law of motion.

Proposition 2 implies that both the steady state, and the transition dynamics, of the distribution of employment across firms are second order in the adjustment friction. Therefore, in the neighborhood of a small adjustment cost, the steady-state firm-size distribution approximately coincides with its frictionless counterpart, and the dynamics of \( h(n) \) are approximately jump. As a result, any gap between the distribution of employment and its frictionless counterpart is closed almost immediately. It is in this precise sense that aggregate outcomes are approximately neutral.

This neutrality result is surprising in a number of respects. It is not anticipated by the general representation of aggregation dynamics in Proposition 1. It holds for any aggregate state \( \Omega \), which includes current and future expectations of market wages. Thus, the neutrality result in Proposition 2 is not the outcome of equilibrium adjustment in wages; it emerges purely from the aggregation of microeconomic behavior. Of course, an implication of the latter is that, since neutrality obtains for any \( \Omega \), \textit{a fortiori} it also will hold for the equilibrium \( \Omega \). Finally, neutrality holds for any (analytic) distribution of past employment \( h_{-1}(n) \). One might expect that the adjustment friction would distort the path of the firm-size distribution the greater the discrepancy between the distribution of inherited employment \( h_{-1}(n) \) and...
its frictionless counterpart $h^* (n)$. Proposition 2 reveals that any such effect is negligible in the presence of a small fixed adjustment cost.

The key to understanding the neutrality result can be traced to a symmetry property in the distributional dynamics of $h (n)$. To see this, it is helpful to rewrite the law of motion for $h (n)$ in equation (9) more directly in terms of its constituent flows as

$$\Delta h (n) = \Pr (\text{adjust to } n) h^* (n) - \Pr (\text{adjust from } n) h_{-1} (n). \quad (14)$$

To see how this sheds light on the source of approximate neutrality, imagine a small fixed adjustment cost is introduced into an otherwise frictionless environment. At any instant of time, the adjustment cost reduces the outflow of mass from any given level of employment $n$, but also reduces the mass of firms which find it optimal to adjust to that level of employment. For small frictions, we show that these two forces are symmetric, leaving the distribution approximately equal to its frictionless counterpart along the transition path.

It is possible to illustrate this argument more formally if we again assume i.i.d. productivity shocks. Recall that, relative to the frictionless case, the introduction of an adjustment cost reduces the outflow of mass from $n$ by

$$h_{-1} (n) (G [U (n)] - G [L (n)]). \quad (15)$$

Among firms positioned at $n$, a share $G [U (n)] - G [L (n)]$ of firms choose not to adjust. Likewise, the inflow of mass to $n$ is reduced at each instant, relative to the frictionless case, by

$$h^* (n) (H_{-1} [L^{-1} X (n)] - H_{-1} [U^{-1} X (n)]). \quad (16)$$

Of the mass $h^* (n)$ of firms for whom $n$ is the desired level of employment, a share of these firms equal to $H_{-1} [L^{-1} (X (n))] - H_{-1} [U^{-1} (X (n))]$ will choose not to adjust. Noting the form of the adjustment triggers in Lemma 1, a second-order approximation to each of the latter expressions around the frictionless optimum reveals that the reductions in both flows converge in the presence of a small adjustment cost, and are approximated by

$$2h_{-1} (n) h^* (n) \tilde{\gamma} [X (n)] \sqrt{C}. \quad (17)$$

---

For instance, from Lemma 1 we can write $G [U (n)] \approx G [X (n)] + g [X (n)] \gamma (n) \sqrt{C} + \frac{1}{2} g' [X (n)] \gamma (n)^2 C$. Following a similar logic for $G [L (n)]$ and differencing yields that $G [U (n)] - G [L (n)] \approx 2g [X (n)] \gamma (n) \sqrt{C}$. Likewise, noting from Lemma 2 that the mandated density $h^* (n)$ approximates its frictionless counterpart, and is thus independent of $G$, we can write $H^* [L^{-1} X (n)] - H^* [U^{-1} X (n)] \approx 2h^* (n) \tilde{\gamma} (X (n)) \sqrt{C}$. The result (17) then follows from the fact that $h^* (n) = g [X (n)] X' (n)$ and $\gamma (n) = X' (n) \tilde{\gamma} [X (n)]$. 

15
It follows that the frictionless mass at any given \( n \) is preserved along the transition path.

A key observation is the dual, symmetric roles played by the densities of inherited and desired employment levels, \( h_{-1}(n) \) and \( h^*(n) \), in equation (17). Holding constant \( h^*(n) \), a large density of inherited employment, \( h_{-1}(n) \), implies that many firms are “trapped” at \( n \), reducing the outflow from that position. But, it also implies that there exist relatively few firms with inherited employment levels sufficiently different from \( n \) that adjusting to \( n \) is optimal, reducing the inflow into the mass. This demonstrates why Proposition 2 holds for any (smooth) initial density: \( h_{-1}(n) \) affects the approximate reduction in outflows and inflows symmetrically. Analogously, holding constant \( h_{-1}(n) \), a greater mass of desired employment at \( n \), \( h^*(n) \), implies that fewer firms find it optimal to adjust away from \( n \), reducing the outflow from that point. But, it also will imply that a greater mass of firms who would prefer to move to \( n \) will be prevented from doing so, reducing the inflow into that mass. These two forces offset, and approximate dynamic neutrality obtains.

3.3 The roles of heterogeneity and two-sided adjustment

To develop understanding of Proposition 2, we highlight two further aspects of the neutrality result that sharpen its interpretation. First, Proposition 2 requires that orders of the adjustment cost greater than \( C \) be small enough to be considered negligible. Under certain restrictions, there is a more precise metric by which the smallness of \( C \) can be evaluated. Consider the family of distributions of idiosyncratic productivity such that \( G(x) = \tilde{G}[(x - \mu) / \sigma] \), where \( \mu \) is a location parameter, and \( \sigma \) a scale parameter that captures dispersion.\(^{21}\) Then, for example, the reduction in the outflow in equation (15) above is given by

\[
h_{-1}(n) \left[ 2\tilde{g} \left( \frac{X(n) - \mu}{\sigma} \right) \gamma(n) \left( \frac{\sqrt{C}}{\sigma} \right) + O \left( \frac{\sqrt{C}}{\sigma} \right)^3 \right].
\]  

Thus, the accuracy of the approximations underlying Proposition 2 hinges on the magnitude of the (square root of the) adjustment cost relative to the dispersion of idiosyncratic shocks \( \sigma \). To see why, recall that the term in brackets is simply the probability of inaction. The approximations obtain if the latter is not very large (though considerable inaction is permitted). The incentive to adjust, in turn, depends on the size of desired adjustments—as governed by the size of changes in productivity—relative to the cost of adjusting. This is captured by \( \sqrt{C} / \sigma \). Alvarez and Lippi (2014) note the same point using different analytical

\(^{21}\) This so-called “location-scale” family of distributions encompasses a variety of commonly-used distributions, including Type-I extreme value, logistic, normal, and exponential distributions, among others.
techniques in a continuous-time Brownian model of price setting. We shall see later that this observation informs our understanding of the quantitative dynamics of the model under alternative calibrations of the adjustment cost $C$ and the dispersion of shocks $\sigma$.\textsuperscript{22}

The second implication of the neutrality result in Proposition 2 that we wish to highlight is the important role of two-sided adjustment—that is, that there exists a positive probability of both hiring and firing workers in each state. To see why this matters, return to the i.i.d. special case, and imagine that the probability of reducing employment $G[L(n)] = 0$ for some employment level $n$, so that adjustment is one-sided upward. The approximations underlying Lemma 2 and Proposition 2 will fail in this case. The reason is that the inaction rate $G[U(n)] - G[L(n)] = G[U(n)]$ ceases to be (approximately) proportional to $\sqrt{C}$, and symmetry is violated.\textsuperscript{23}

Two-sided adjustment fails in our environment only in restrictive cases. Here we highlight two examples. First, in the presence of a lump-sum fixed adjustment cost and a lower bound on the distribution of idiosyncratic shocks, it is possible that the lower adjustment trigger $L(n)$ dips below the lower support of $x$ at small employment levels—(very) small firms will adjust only upward. A second, related example is the case in which employment attrites exogenously at rate $\delta$ in the absence of adjustment. Appendix A shows that Lemma 2 and Proposition 2 remain intact under attrition, provided $\delta$ is not so large that adjustment becomes one-sided (the firm only hires). Again, this will happen only at very small firms, since any further desired reductions in employment can often be carried out via attrition. The extent to which this binds is a quantitative issue to which we return in section 4.

### 3.4 Applications to capital and price adjustment

Our analysis thus far has been cast in the context of a dynamic labor demand problem. We noted earlier, however, that our results apply equally to canonical models of capital and price adjustment. Here, we briefly explain why.\textsuperscript{24} We shall see later that this clear isomorphism aids the comparison of the results noted above with prior literature which spans these related employment, capital and price adjustment problems.

\textsuperscript{22}This formalizes the intuition in Bertola and Caballero (1990) who note that, if the distribution of $x$ degenerates, either all firms do not react to aggregate shocks $p$, or all adjust, a dramatic departure from the frictionless case. In this sense, the extent of productive heterogeneity has to matter for aggregate dynamics.

\textsuperscript{23}Specifically, $G[U(n)] - G[L(n)] = G[U(n)] \approx G[X(n)] + g[X(n)] \gamma(n) \sqrt{C}$ in this case, as opposed to $2g[X(n)] \gamma(n) \sqrt{C}$ in the case of two-sided adjustment.

\textsuperscript{24}As in canonical models of employment, capital and price adjustment, we treat each of these problems in isolation, neglecting any interactions. A limited literature has considered the interaction of employment and capital adjustment (Shapiro, 1986; Dixit, 1997; Eberly and van Mieghem, 1997; Bloom, 2009). Even less work has studied interactions with price rigidities (a notable exception is Reiter, Sveen and Weinke, 2009).
Capital adjustment. Reinterpretation of our results for the case of capital adjustment is especially straightforward. The canonical decision problem faced by a firm is given by:

\[
\Pi (k_{-1}, x; \Omega) \equiv \max_k \left\{ pxF (k) - Rk - C1^\Delta + \beta \mathbb{E} [\Pi (k, x'; \Omega') | x, \Omega] \right\},
\]

(19)

where \( k \) denotes capital, and \( R \) the rental rate on capital.\(^{25}\) By direct analogy to the labor demand case, the aggregate state \( \Omega \) will include the rental rate \( R \), aggregate productivity \( p \), and any information pertaining to their future evolution—in particular, perceptions of the current and future distributions of capital. The isomorphism is thus clear: one can pass from (1) to (19) simply by replacing \( n \) with \( k \), and \( w \) with \( R \). It follows that the equilibrium outcome also will coincide approximately with the frictionless equilibrium.

It is worth re-emphasizing here that the Appendix establishes that approximate neutrality also holds in the presence of depreciation, which is especially applicable to the case of capital adjustment. Depreciation lowers all three policy functions, \( L (n) \), \( X (n) \) and \( U (n) \), in approximately the same way: Firms are more likely to adjust upward, choose higher levels of \( k \) conditional on adjusting, and are less likely to adjust downward. This preserves the symmetry of the problem that underlies the neutrality result. Note that the symmetry required for neutrality therefore does not require symmetry of adjustment—neutrality holds in this case even though firms are more likely to adjust upward than downward.

Price adjustment. The problem of price setting under fixed menu costs has a similar structure. Consider a firm facing an isoelastic demand schedule of the form \( y = (p/P)^{-\epsilon} Y \), where \( p \) is the firm’s price; \( P \) is the aggregate price level; \( Y \) is real aggregate output; and \( \epsilon > 1 \) is the elasticity of product demand. If the firm operates a linear production function \( y = xn \), and faces a market wage \( w \), then one can re-cast the firm’s problem as one of choosing the transformed price \( q \equiv p^{-\epsilon} \):

\[
\Pi (q_{-1}, x; \Omega) \equiv \max_q \left\{ Z \alpha - Z \left( \frac{w}{x} \right) q - C1^\Delta + \beta \mathbb{E} [\Pi (q, x'; \Omega') | x, \Omega] \right\},
\]

(20)

where \( \alpha \equiv (\epsilon - 1) / \epsilon \in (0, 1) \), and \( Z \equiv P^\epsilon Y \) is a measure of nominal aggregate demand. Again, the form of (20) has a similar structure to the baseline model of section 1, but where the aggregate state \( \Omega \) is now comprised of the market wage \( w \), nominal aggregate demand \( Z \), and perceptions of current and future distributions of prices. Once again, then, the aggregation and neutrality results of Propositions 1 and 2 apply to this pricing problem.

\(^{25}\) A standard user cost argument implies that the rental rate \( R \) can in turn be related to the price of capital \( P_k \) according to \( R \equiv P_k - \beta (1 - \delta) \mathbb{E} [P'_k] \).
3.5 Relation to the literature

It is instructive to compare our neutrality result in Proposition 2 with related results in the prior literature. Caplin and Spulber (1987) were the first to note the possibility of aggregate neutrality in the presence of lumpy microeconomic adjustment in a related pricing problem. They consider a very simple environment without idiosyncratic shocks and one-sided $S$s adjustment. Their ingenious result is that an invariant uniform cross-sectional distribution will be preserved in such an economy, and that aggregate outcomes are unaffected by the adjustment cost.

Like ours, Caplin and Spulber’s result arises from a form of symmetry in the model’s distributional dynamics: Common shocks move all firms in the same direction in the $S$s band, and firms induced to adjust at the bottom of the uniform distribution exactly replace those displaced at the top of the distribution. Proposition 2 shows that the Caplin and Spulber insight can be generalized approximately to an environment with quite general idiosyncratic heterogeneity, and two-sided adjustment.26

Golosov and Lucas (2007) add precisely the ingredients of our baseline model to Caplin and Spulber’s problem. In their numerical solution of the model, they indeed find very small effects of money on aggregate output. Golosov and Lucas suggest that the robustness of Caplin and Spulber’s neutrality result stems from a property of the $S$s models referred to as the selection effect. The idea is that firms that adjust are those that wish to change their price by a lot. Hence, the claim is that, although many firms do not adjust, the aggregate adjustments are large, and neutrality obtains.

The notion of a selection effect from Golosov and Lucas is formalized in the symmetry result underlying Proposition 2. To see this, recall the symmetric effect of $h_{-1}(n)$ on the inflows to and outflows from $n$. As we noted, if $h_{-1}(n)$ is large, then many firms are “trapped” at $n$, and outflows from this position are reduced. But, it also means there are many firms near $n$. These firms are less likely to select into $n$ if it is their desired choice, since the small increase in profits does not outweigh the adjustment cost $C$. This latter, symmetric reduction in the inflows to $n$ is an expression of the selection effect. Hence, our characterization of symmetry in the distributional dynamics formalizes the intuition gleaned from Golosov and Lucas’ numerical analysis.

26Caballero and Engel (1991, 1993) retain the assumption that adjustment is one-sided, but allow the rate of increase to vary across units. They show that, if the initial difference between actual and desired prices—the “price gap”—is uniformly distributed about zero, this distribution is preserved under an $S$s adjustment policy. A form of symmetry is also at work here. Since idiosyncratic shocks are assumed to be uncorrelated with initial gaps, and since gaps are uniformly distributed, the outflow from a high gap is offset by the inflow from a low gap.
A more recent literature has emphasized the role of equilibrium adjustment in market prices in unwinding the aggregate effects of lumpy adjustment (see Khan and Thomas, 2008; Veracierto, 2002; and House, 2008). It is important to note that the neutrality result in Proposition 2 is quite distinct from these channels. Specifically, Proposition 2 suggests that approximate neutrality holds for any aggregate state—which includes the wage—that is, regardless of aggregate price movements. What is at the heart of Proposition 2 is an aggregation result that emerges from the symmetry in the distributional dynamics of $h(n)$.

Finally, recent numerical analyses have found that deviations from frictionless dynamics can be more significant than implied by Proposition 2, if market prices are fixed (King and Thomas, 2006; Khan and Thomas, 2008). Our results suggest that these deviations arise from disruptions of symmetry. In the next section, we show that this can occur when the adjustment cost is large enough relative to idiosyncratic dispersion to violate the approximations underlying Proposition 2. We now turn to these, and related, quantitative issues.

4 Quantitative Analysis

A natural question in the light of Proposition 2 is whether plausible parameterizations of the model imply aggregate dynamics that resemble the approximate results of Proposition 2, or the more general results of Proposition 1. We address this question in section 4.1 by parameterizing the model using conventional estimates. We then study the effects of alternative calibrations of the parameters of the model in section 4.2, and use this to contrast our results with recent quantitative analyses in the related literature. Finally, in section 4.3 we illustrate analytically how one particular extension of the baseline model can generate aggregate non-neutralities by breaking the symmetry underlying Proposition 2.

4.1 Baseline quantitative analysis

The baseline parameterization we use is summarized in Table 1. The numerical model is cast at a quarterly frequency. We adopt the widespread assumption that the production function takes the Cobb-Douglas form, $F(n) = n^\alpha$, with $\alpha < 1$. The returns to scale parameter $\alpha$ is set equal to 0.64 based on estimates reported in Cooper, Haltiwanger and Willis (2005, 2007). This also is similar to the value assumed by King and Thomas (2006). The discount factor $\beta$ is set to 0.99, which is the conventional choice for a quarterly model.

Of course, this does not preclude that equilibrium price adjustment can weaken the effects of lumpy adjustment on aggregate dynamics in cases where the approximations underlying Proposition 2 do not hold.
The magnitude of the adjustment cost is based on estimates reported in Cooper, Haltiwanger and Willis (2005) and Bloom (2009). Cooper et al. (2005) estimate a model similar to the one described above using plant-level data from the Census’ Longitudinal Research Database. In one of their better-fitting specifications, they estimate a cost of adjustment equal to 8 percent of quarterly revenue (see row “Disrupt” in their Table 3a). Using annual Compustat data, Bloom (2009) finds nearly the same result, once it is converted to a quarterly frequency (see column “All” in his Table 3). Based on this, we set the adjustment cost parameter $C$ to replicate these estimates.\footnote{Bloom’s and Cooper et al.’s main estimates are derived from a setup whereby the fixed cost is scaled by firm revenue. In a version of their model with a lump-sum fixed cost, Cooper et al. estimate a smaller adjustment cost than our baseline choice. In this sense, we have erred on a side of a larger adjustment cost.}

It turns out that this value of $C$ also implies an average frequency of adjustment that is comparable to what is observed in U.S. establishment-level data. In particular, it yields an estimate of the average quarterly probability of adjusting of 56 percent, as compared to 48.5 percent in U.S. data.\footnote{This estimate is available from the BLS Business Employment Dynamics program. See http://www.bls.gov/bdm/bdsoc.htm. We take the average over the full sample, 1992q3 to 2013q2.}

Idiosyncratic and aggregate shocks are assumed respectively to evolve according to the common assumption of geometric AR(1) processes,

\begin{align}
\log x' &= \mu_x + \rho_x \log x + \epsilon'_x, \quad \text{and} \\
\log p' &= \mu_p + \rho_p \log p + \epsilon'_p, \quad \text{(21)}
\end{align}

where the innovations are independent normal random variables: $\epsilon'_x \sim N(0, \sigma^2_x)$, and $\epsilon'_p \sim N(0, \sigma^2_p)$. This baseline parameterization in (21) is again informed by Cooper, Haltiwanger, and Willis (2005, 2007), since they recover estimates within related labor demand models. Their estimates of $\sigma_x$ range from about 0.2 (in their 2007 paper) to 0.5 (in their 2005 paper). We split the difference and set $\sigma_x = 0.35$.\footnote{Cooper et al. (2005) estimate four versions of a dynamic labor demand problem. Three of their estimates of $\sigma_x$ cluster around 0.5; the other is 0.22. Cooper et al. (2007) also present four sets of estimates of $\rho_x$ and $\sigma_x$, expressed at a monthly frequency, that correspond to four model variants (see their Table 5). These yield estimates of the quarterly standard deviation $\sigma_x$ centered about 0.2.}

However, it has been noted that these papers’ estimates of $\rho_x$, most of which are below 0.5, appear rather low relative to other estimates in the literature; Cooper and Haltiwanger (2006) and Foster, Haltiwanger, and Syverson (2008) each recover estimates of $\rho_x$ near 0.95.\footnote{Cooper and Haltiwanger’s (2006) estimate from annual data implies a quarterly value of $\rho_x$ equal to 0.885$^{1/4} = 0.97$. Foster et al. (2008) estimate both productivity (“physical TFP”) and product demand processes. The implied quarterly autoregressive parameters are, respectively, 0.943 and 0.976.} Again, we split the difference and set $\rho_x = 0.7$, close to the midpoint of this wider range of estimates.\footnote{This value is also close to that estimated in Abraham and White (2006) using U.S. manufacturing data.} This baseline parameterization is
comparable to that used in Bachmann’s (2013) analysis of non-convex adjustment costs.

The parameters of the process of aggregate shocks, $\rho_p$ and $\sigma_p$, are calibrated so that the model approximately replicates the persistence and volatility of (de-trended) log aggregate employment. Using postwar quarterly time series on private payroll employment, and de-trending using an HP filter with smoothing parameter $10^5$, we compute an autocorrelation coefficient of 0.96 and a standard deviation of 0.025. Values of $\rho_p = 0.95$ and $\sigma_p = 0.015$ are roughly consistent with these moments (see Table 1). We do this because our goal is not to explain the volatility of aggregate employment, but to compare model outcomes within an environment that is economically relevant. One way of doing that is to generate aggregate outcomes that are comparable to what we observe in the data.

Lastly, as noted at the conclusion of section 1, we have generalized the analysis of sections 2 and 3 to allow for worker attrition. Accordingly, we have incorporated a constant rate of attrition, $\delta$, into our quantitative analysis. To calibrate $\delta$, we use the simple average of the quarterly quit rate from the Job Openings and Labor Turnover Survey. This is 6 percent.

As stressed in Proposition 2, approximate neutrality obtains for any given aggregate state, which includes the wage, and thus is not an outcome of equilibrium price adjustment. It is, instead, an aggregation result that relies only on the symmetry in the distributional dynamics. To emphasize this point, we simulate the model for a fixed wage. The latter is chosen to induce an average firm size of 20, which is in line with evidence from the Census’ Business Dynamics Statistics.\(^\text{33}\) In Appendix B, we discuss how to implement the model in general equilibrium and present impulses responses in this case. The results for the baseline parameterization are virtually identical to what we present here.

Since the wage is fixed, firms do not need to forecast future wages. This means, in turn, that they do not need to forecast future employment distributions. Therefore, the aggregate state $\Omega$ is summarized completely by aggregate productivity $p$, and the optimal policy functions take the simple form $L(n; p), X(n; p), \text{ and } U(n; p)$. As we noted in section 1, a positive innovation to aggregate productivity $p$ shifts these functions downward—for a given level of idiosyncratic productivity, a firm is more likely to hire, less likely to fire, and will select a higher level of employment conditional on adjustment. Thus, the evolution of aggregate productivity $p$ induces shifts in the policy function, which, via the law of motion (9), trace out the evolution of the distribution of employment and thereby aggregate employment.

The results of this exercise under the baseline calibration are illustrated in Figures 2 and 3. We begin in Figure 2 by analyzing the properties of the steady-state distribution

\(^{33}\)See http://www.census.gov/ces/dataproducts/bds/data_estab.html. We compute the average firm size over the full sample for the years 1977 to 2011.
of employment that would be attained in the absence of aggregate shocks. The latter is compared to two reference distributions. The first is the frictionless distribution. The second is the distribution induced by a myopic labor demand policy, in reference to Lemma 2.

Figure 2 reveals that the steady-state distribution of employment mimics closely its myopic and frictionless counterparts at virtually all employment levels. As foreshadowed by the discussion in section 3.3 highlighting the important role of heterogeneity and two-sided adjustment, any deviations that do emerge are restricted to very small firm sizes of fewer than two workers. Moreover, these discrepancies are very small in practice. Figure 2 thus confirms that the neutrality of the steady-state distribution implied by Proposition 2 is a prediction upheld by conventional parameterizations of an employment adjustment problem.

In Figure 3 we turn to the dynamic implications of the model. Panel A presents the impulse response of aggregate employment to a one-percent positive innovation to aggregate labor productivity $p$ implied by the baseline parameterization, and contrasts it with its frictionless ($C = 0$) and myopic ($\beta = 0$ and $C > 0$) counterparts. The differences between the impulse responses are so small as to be almost imperceptible. Thus, the prediction of approximate dynamic neutrality in Proposition 2 is not merely a theoretical curiosity; it holds under an empirically-relevant set of parameters.

The source of this approximate neutrality is illustrated in panel B of Figure 3. This exercise is informed by the emphasis of Proposition 2 on the symmetry of the effects of adjustment frictions on the flows in and out of the mass at each employment level. In particular, rearranging the identity in equation (14), multiplying through by $n$, and integrating yields the following description of the relation between actual and frictionless aggregate employment:

$$N = N^* + \int n \left[ \text{reduction in outflows} \right] dn - \int n \left[ \text{reduction in inflows} \right] dn. \quad (23)$$

Here $N \equiv \int nh(n) \, dn$ is aggregate employment in the baseline (forward-looking) model and $N^* \equiv \int nh^*(n) \, dn$ is its frictionless counterpart. The aggregate effects of the adjustment cost are thus mediated by the final two terms on the right-hand side of (23). These represent the employment-weighted reductions, relative to the frictionless model, in the flows in and out of each employment level. Panel B of Figure 3 plots the impulse responses of these two terms, normalized by pre-impulse aggregate employment.$^{34}$

Two results emerge from the exercise in Figure 3B. First, the aggregate reductions in the inflows and outflows induced by the fixed cost are substantial. At their peak, each amounts

---

$^{34}$Formally, these are calculated by generalizing the i.i.d. case, expressed in (15), to account for persistent shocks. For example, the reduction in outflows is computed as $h_{-1} (n) (G [U (n) | n] - G [L (n) | n]).$
to about 20 percent of steady-state employment. In this sense, the fixed adjustment cost
does disrupt significantly the flows to and from each point along the distribution. Second,
as predicted by the neutrality result in Proposition 2, the effect of the adjustment cost
on the inflows is almost perfectly offset by its effect on the outflows. At no point does
the difference exceed 0.6 of one percent. Moreover, the two series move in tandem. This
illustrates the symmetry in the distributional dynamics that underlies the approximately
frictionless aggregate dynamics in the model.

Interestingly, these quantitative results dovetail with recent literature on dynamic factor
demand that has solved numerical models of fixed adjustment costs under specific para-
metric assumptions. Our finding that aggregate dynamics are approximately invariant with
respect to the fixed cost mirrors the findings of Cooper, Haltiwanger and Power (1999) and
Cooper and Haltiwanger (2006), who find empirically that, in the case of capital adjustment,
aggregation smooths away much of the effect of the adjustment friction.

4.2 Sensitivity analysis

In this section, we investigate the robustness of the results presented thus far. We consider
plausible variations on the baseline parameterization based on six experiments.

Raising $C$ relative to $\sigma_x$. The first two experiments investigate the effects of alternative
choices of the adjustment cost $C$ and the dispersion of idiosyncratic shocks $\sigma_x$. These exer-
cises are motivated by the discussion of section 3.3, which highlights the crucial role of the
magnitude of $C$ relative to $\sigma_x$ in the neutrality result in Proposition 2.

Panel A of Figure 4 considers the effects of increasing $C$ so that the adjustment cost is
16 percent of revenue, on average, across firms. This corresponds to a two-standard error
increase above Bloom’s (2009) estimate. Likewise, in panel B of Figure 4, we lower the
standard deviation of innovations to idiosyncratic productivity $\sigma_x$ to 0.2, in line with the
lower end of estimates in the literature surveyed in section 4.1.\(^{35}\) As before, we compare
these impulse responses to their frictionless counterparts, and illustrate the corresponding
reductions in the constituent flows outlined in equation (23).\(^{36}\)

Both of these experiments lower rates of adjustment: Average quarterly adjustment
probabilities are 44 percent in the parameterization underlying Figure 4A, and 28 percent in
that underlying Figure 4B. This greater degree of inaction is in turn reflected in the impulse

\(^{35}\)To hold all else equal, we adjust $C$ so that it continues to equal 8 percent of revenue, on average.

\(^{36}\)To avoid clutter, in what follows we omit the impulse responses generated by the myopic model. In each
case, these are very similar to the impulse responses in the baseline model.
responses in Figures 4A and 4B. The latter in particular reveals a modest hump-shape, with a peak response after just one quarter, and almost frictionless dynamics thereafter. The contrast with Figure 3 is consistent with our interpretation of Proposition 2, which revealed that symmetry is likely to fail if productive heterogeneity is more limited relative to the adjustment friction. But, the magnitudes of the deviations remain small.

Matching the frequency and size of adjustments. The latter experiments have counterfactual implications for rates of employment adjustment, however. As noted above, the empirical rate of employment adjustment is much higher than that underlying Figure 4B, at 48.5 percent in U.S. establishment-level data. For this reason, in our third experiment we explore the effects of calibrating the adjustment cost $C$ and the dispersion of idiosyncratic shocks $\sigma_x$ to target two salient moments of the cross-establishment distribution of employment growth: the average quarterly frequency of adjusting of 48.5 percent; and the average absolute quarterly log change in employment among adjusters, which is 0.31.\(^{37}\) This exercise significantly reduces the adjustment cost to just 0.36 percent of average quarterly revenue, as well as the degree of idiosyncratic dispersion $\sigma_x$, which falls to 0.08.

Panel C of Figure 4 presents the results of this experiment. Reiterating the important role of the rate of adjustment in the approximations underlying Proposition 2, Figure 4C reveals that this alternative calibration strategy largely restores the neutrality result noted in the baseline case in Figure 3: the impulse response is almost indistinguishable from the frictionless analogue. The message of this experiment is that Proposition 2 is quantitatively relevant in a calibration that replicates key aspects of the cross section of employment growth.

Varying idiosyncratic persistence, $\rho_x$. We noted earlier that leading estimates of the persistence of idiosyncratic productivity shocks $\rho_x$ vary widely across studies. A common intuition is that firms should adjust less aggressively to idiosyncratic shocks if productivity is more transitory in order to position employment so it is optimal given expected future reversion to mean in productivity. However, the myopic approximation in Lemma 2 suggests the payoff to this foresight is small. For this reason, Proposition 2 suggests that the lack of empirical consensus over $\rho_x$ is inessential to the presence or otherwise of approximate aggregate neutrality—the result holds independently of $\rho_x$. Motivated by this, in a fourth experiment we consider the effects of lowering $\rho_x$ to 0.4 (in line with the majority of Cooper et al.’s estimates), and of raising $\rho_x$ to 0.9 (closer to the estimates of Foster et al.). Panel D

\(^{37}\)Thanks to David Ratner, who provided these estimates from BLS Business Employment Dynamics (BED) microdata. The latter record quarterly employment for nearly 75 percent of U.S. establishments.
of Figure 4 illustrates the results and confirms the predictions of Proposition 2: Changing $\rho_x$ has almost no effect on the impulse response of aggregate employment, which continues to track its frictionless path.

**Stochastic adjustment costs.** Our baseline model assumes a lump-sum fixed cost, $C$. A common alternative specification adopted in recent literature is one whereby the adjustment cost is drawn each period from a given distribution.\(^{38}\) It is straightforward to incorporate such stochastic fixed costs into the above model and to (re-)prove our propositions. Suppose that fixed costs are drawn from a distribution with upper support, $\bar{C}$. If $\bar{C}$ is small (in the sense discussed in section 3), then the approximation to the adjustment triggers in Lemma 1 can be applied for any $C < \bar{C}$. Moreover, under this assumption, the order-of-magnitude argument behind the optimality of myopia in Lemma 2 also is preserved. As a result, one can adapt the approach of section 3 to show that, to a first-order approximation, the neutrality result in Proposition 2 remains intact.

To pursue this argument further, Figure 4E plots the implied impulse responses for aggregate employment from a version of the baseline model in which firms take i.i.d. draws of fixed costs from a uniform distribution bounded below by 0 and above by $\bar{C}$, as in King and Thomas (2006). All other parameters in the baseline case are retained. We consider two parameterizations of $\bar{C}$. The first sets $\bar{C}$ to the value of the lump-sum fixed cost used in the baseline calibration. The second chooses $\bar{C}$ so that the average probability of adjusting coincides with its value in the baseline calibration. The results of Figure 4E confirm that the presence of stochastic fixed adjustment costs *per se* has little effect on the baseline results.\(^{39}\)

**Size-dependent adjustment costs.** A second alternative specification of adjustment costs used in recent literature has been to scale these costs by some measure of firm size, so that firms do not outgrow the friction.\(^{40}\) Two common approaches have been implemented. First, Caballero and Engel (1999) and Gertler and Leahy (2008) scale the adjustment cost to


\(^{39}\)That is not to say that the presence of stochastic adjustment costs may not play a role under different parameterizations of the model. For example, Gourio and Kashyap (2007) highlight the importance of the shape of the distribution of adjustment costs for the aggregate dynamics of investment. However, their model abstracts from the presence of idiosyncratic heterogeneity ($\sigma_x = 0$). Figure 4E suggests that such effects are not large in conventional parameterizations of employment adjustment models in which idiosyncratic dispersion is estimated to be significant.

\(^{40}\)However, the probability of adjusting employment in BLS Business Employment Dynamics micro data does increase in establishment size. One interpretation is that it is consistent with a lump-sum friction. By contrast, formalizations of size-dependent costs typically imply that firms are never large relative to the adjustment cost, and thus fail to replicate this fact.
be proportional to frictionless revenue, \( C = cR(x) \). In a second specification, the adjustment cost is modeled as a share of current revenue, \( C = cxF(n) \). This is the specification used in Cooper et al. (2005, 2007), Bloom (2009), and Bachmann (2013). Note that these cases imply a certain asymmetry to the adjustment cost function.

Consider first the simpler case of \( C \equiv cR(x) \). We show in Lemma 4 in the Appendix that approximate neutrality continues to hold under this specification of size-dependent frictions. Figure 4F confirms this prediction. It presents the implied impulse response in the case where \( c \) is set to replicate the average adjustment rate in the baseline parameterization illustrated in Figure 3. It is almost indistinguishable from the frictionless response.

This extended result in turn aids interpretation of the more complicated case in which \( C = cxF(n) \). The latter is increasing in the choice of employment. It follows that the adjustment cost distorts the optimal level of employment conditional on adjusting, \( n = X^{-1}(x; c) \), because it acts like a tax on increases in \( n \). Thus, the key difference in this model is that the distribution of mandated employment implied by this distorted reset policy will diverge from its frictionless analogue. All other features resemble the case above where \( C = cR(x) \). Therefore, neutrality obtains with respect to the path of mandated, but not frictionless, aggregate employment. However, the deviations from the frictionless path are quite small, as shown in Figure 4F. Relative to the simpler size-dependent case, a slight deviation emerges on impact, but this is subsequently eliminated.

### 4.3 Relation to the literature

What emerges from the foregoing quantitative analysis is that the presence of a fixed adjustment cost has, at most, only a modest effect on aggregate dynamics under reasonable parameterizations, even in the absence of adjustment of market prices. As in Proposition 2, the source of these limited effects can be traced to the symmetric role of the adjustment cost in reducing the flows in and out of each position in the cross section. And, where (small) deviations in aggregate dynamics do arise, it is in parameterizations that imply rates of adjustment significantly lower than those seen in microdata on employment.

These observations share parallels in prior literature based on numerical work. For instance, King and Thomas (2006) document deviations of aggregate dynamics with respect to the frictionless case when market prices are fixed, as they are in the simulations reported above. Gourio and Kashyap (2007) report similar quantitative findings in their analysis of a related investment problem. For simplicity, however, the sole source of heterogeneity in both of these analyses is (modest) variation in the form of a stochastic fixed cost of adjust-
ment; both studies abstract entirely from productive heterogeneity, implicitly imposing that $\sigma_x = 0$. The foregoing analysis thus suggests that the non-neutralities found in these earlier studies are a consequence of the assumed absence of idiosyncratic heterogeneity.\footnote{Bachmann (2013) also finds that a fixed adjustment cost model induces sluggish dynamics in aggregate employment. While his model allows for idiosyncratic risk comparable to that used in this paper, his calibration still implies a comparatively low adjustment rate.}

Important precedents in prior literature do allow for productive heterogeneity, however. Khan and Thomas (2008) provide a calibration of a related investment model that success-\footnote{For example, Khan and Thomas’ calibration implies that 75 percent of plants would not adjust their capital stock in a given year, but for the fact that their model exempts very small adjustments from the adjustment cost.}fully confronts several features of the data on plant-level investment. The implied dispersion in productivity $\sigma_x$ is such that adjustment rates are significantly lower than in our baseline case above.\footnote{These estimates are taken from Klenow and Malin’s Table 7. The lower end of this range (7 months) is found by comparing “like” prices. This approach retains observations on sales-related price changes only if the current sale price differs from the most recent sale price; sale and non-sale prices are never compared. Mean (median) duration rises to 8 (6.9) months if all sales-related price changes are dropped.} As foreshadowed by the interpretation of Proposition 2, and the quantitative analysis in Figure 4B, Khan and Thomas find that deviations emerge between frictionless dynamics and the behavior of aggregate capital in the presence of the adjustment costs, if market prices are fixed. This suggests that calibrations similar to that summarized in Figure 4B may be relevant to the case of capital adjustment.

Our own analysis suggests that higher adjustment rates are more relevant for the case of labor demand, and that these in turn imply aggregate dynamics almost indistinguishable from their frictionless counterpart. Interestingly, our results also suggest that estimates of the frequency of price adjustment imply very limited non-neutrality. The recent survey of Klenow and Malin (2011) suggests that, after omitting many sales-related price changes, the mean (median) duration of prices is about 7 (5.9) months. If price changes coinciding with product substitutions are excluded, the mean (median) duration rises to 10 (8.3) months.\footnote{These estimates are taken from Klenow and Malin’s Table 7. The lower end of this range (7 months) is found by comparing “like” prices. This approach retains observations on sales-related price changes only if the current sale price differs from the most recent sale price; sale and non-sale prices are never compared. Mean (median) duration rises to 8 (6.9) months if all sales-related price changes are dropped.}
4.4 Generating non-neutralities: An analytical illustration

We close this section by highlighting how the analytical framework provided in this paper can help elucidate the sources of non-neutralities. An influential strand of recent research has argued that the form of idiosyncratic shocks plays a crucial role in shaping the aggregate effects of lumpy microeconomic adjustment. In particular, Gertler and Leahy (2008) and Midrigan (2011) have studied environments in which idiosyncratic shocks evolve according to a compound Poisson process whereby individual firms receive a shock with probability \(1 \pm \lambda\) each period. Interestingly, they find that this departure gives rise to persistent aggregate dynamics, in contrast to the results of previous sections of this paper.\(^{44}\)

In what follows, we show that the analysis and intuition of sections 2 and 3 provide a novel perspective on the origins of this result. In particular, we are able to trace this result analytically to a clear violation of symmetry in the distributional dynamics.

For clarity, consider the case in which idiosyncratic shocks are conditionally i.i.d. That is, with probability \(1 \pm \lambda\) each period firms receive an independent draw \(x'\) from a distribution function \(G(x')\), while with probability \(\lambda\) no idiosyncratic shock arrives and \(x' = x\).

As in section 3, our aim is to approximate the reductions in the flows in and out of the mass \(h(n)\) relative to a frictionless world in which all firms adjust every period.\(^{45}\) Note that these flows essentially are unchanged for the set of firms that receive an idiosyncratic shock. What is different is that there exists a mass of firms that receive no idiosyncratic shock, but may adjust to aggregate shocks.

In their model of menu costs, Gertler and Leahy (2008) show that almost none of the latter firms in fact adjusts in the presence of plausibly small aggregate disturbances. The same is true of our model. To understand why, it is helpful first to imagine the model in the absence of aggregate shocks. In that case, a firm that receives no idiosyncratic shock has no reason to adjust: If their current productivity \(x = x_{-1}\) lies outside of the inaction region \([L(n_{-1}), U(n_{-1})]\), then it must also have done in the past, and the firm already will have adjusted. All that changes in the presence of aggregate shocks is that the current period’s adjustment triggers may differ from the previous period’s, inducing some firms on the margin to adjust. When aggregate shocks are small relative to the inaction region, the

\(^{44}\)An earlier version of Midrigan (2011) used an different shock process, but which nonetheless exhibited a discontinuity at zero. In effect, this gives rise to an atom in the density function, just as in the case studied here. It should also be noted that our analysis abstracts from other dimensions of Midrigan’s (2011) model, such as the presence of multi-product firms and the associated economies of scope in price adjustment. Midrigan shows that the latter also contribute to non-neutralities.

\(^{45}\)A subtle but important point is that, even though firms receive idiosyncratic shocks with probability \(1 \pm \lambda < 1\), they still adjust every period in a frictionless world due to the presence of aggregate shocks.
latter measure of firms will be small.\footnote{By the same token, among firms with \( x = x_{-1} \), a discrete mass will have adjusted in the past and will inherit an employment level of \( n_{-1} = X^{-1}(x_{-1}) \). It follows that aggregate shocks that shift the reset function \( X(\cdot) \) enough to induce even these firms to adjust in the current period will induce a discretely-large fraction of firms to adjust. Thus, large aggregate shocks will be more likely to induce neutrality in the presence of Poisson shocks. Karadi and Reiff (2012) investigate this possibility in more detail.}

It follows that the reduction in the outflow from \( n \) relative to the frictionless case is approximated by \( h_{-1}(n) \{ (1 - \lambda) (G[U(n)] - G[L(n)]) + \lambda \} \): Of the \( 1 - \lambda \) firms that receive an idiosyncratic shock, a fraction \( G[U(n)] - G[L(n)] \) will not adjust away from \( n \); and a share \( \lambda \) receives no idiosyncratic shock and also does not adjust. Similarly, the reduction in the inflow into \( n \) is approximated by \( h^*(n) \{ (1 - \lambda) (H_{-1}[L^{-1}X(n)] - H_{-1}[U^{-1}X(n)]) + \lambda \} \).

Comparison of the latter with the analysis of the continuous-shock case in section 3 reveals the mechanism at the heart of the persistence induced by the Poisson model. As in section 3, the reductions in the flows associated with firms that receive idiosyncratic shocks approximately cancel in the presence of a small fixed adjustment cost. What remain are the terms associated with firms that have not received an innovation to \( x \). Crucially, these flows do not cancel. As a result, the implied approximate aggregate dynamics are\footnote{Recall that the frictionless law of motion is \( \Delta h(n) = -[h_{-1}(n) - h^*(n)] \). We have shown that the outflows from \( n \) are depressed relative to the frictionless case by \( \lambda h_{-1}(n) \) and the inflows to \( n \) are depressed by \( \lambda h^*(n) \). Thus, we can amend the frictionless law of motion to obtain \( \Delta h(n) \approx -[h_{-1}(n) - h^*(n)] + \lambda h_{-1}(n) - \lambda h^*(n) \), which yields the expression in the main text.}

\[
\Delta h(n) \approx - (1 - \lambda) [h_{-1}(n) - h^*(n)].
\]

What emerges, then, is that aggregate dynamics in the presence of Poisson shocks are approximated by a pure partial-adjustment process, with convergence rate equal to the probability of receiving an idiosyncratic shock, \( 1 - \lambda \). Equivalently, the model will behave like a Calvo model in which the exogenous probability of adjusting is set to \( 1 - \lambda \). Since the latter is independent of the level of employment \( n \) (in contrast to the continuous-shock case studied above), it follows that aggregate employment will inherit precisely the same partial-adjustment dynamics. Hence, we expect persistent, hump-shaped impulse responses.

To illustrate this point, we calibrate the model with Poisson idiosyncratic shocks and compute the impulse response of aggregate employment to an aggregate productivity innovation. To maximize similarity with the benchmark model, we leave virtually all of the structural parameters unchanged, and modify the adjustment cost to guarantee that it remains equal to 8 percent of revenue on average. Since any \( \lambda > 0 \) necessarily lowers the probability of adjusting \( ceteris paribus \), however, this calibration will not match the baseline inaction rates. Instead, we compare the Poisson case with a calibration of the benchmark
model that implies a comparably small adjustment probability. We find that \( \lambda = 0.45 \) induces a probability of adjusting in the Poisson model that is similar to that in the low-\( \sigma_x \) parameterization of the benchmark model depicted in Figure 4B.

Figure 5 illustrates the results. Consistent with the results of Gertler and Leahy (2008) and Midrigan (2011), one can clearly discern much more persistent aggregate dynamics in this case, with employment converging to its frictionless counterpart after five quarters.\(^{48}\) Moreover, the persistence cannot be attributed to a lower average adjustment rate—the low-\( \sigma_x \) case in Figure 4B induces a similar adjustment rate but exhibits much less propagation.

Rather, the persistence is closely linked to the above intuition for the approximate partial-adjustment nature of the model’s dynamics in the presence of Poisson shocks. To emphasize this point, Figure 5 also plots the path of aggregate employment directly from the approximate pure partial-adjustment result in (24) as a point of comparison with the model-generated path. Remarkably, the two paths are almost indistinguishable, suggesting that the approximate analysis above indeed provides a very good guide to the behavior of the model.

The source of this result can be traced to a violation of the symmetry noted in section 3. There we highlighted the dual, symmetric roles of the distributions of inherited and desired employment, \( h_{-1}(n) \) and \( h^*(n) \), in delivering aggregate neutrality in the presence of continuous shocks. For instance, while it seems clear that \( h_{-1}(n) \) is indicative of the mass of firms that is deterred from adjusting away from \( n \), a more subtle point is that it also contributes to the size of the reduction in the probability of adjusting to \( n \). The reason is that firms whose initial employment is near \( n \) (mass in the neighborhood of \( h_{-1}(n) \)) do not find it optimal to adjust to that position. Hence, what underlies this latter, symmetric effect is the fact is that a firm’s propensity to adjust (to \( n \)) depends on its initial size. The model with Poisson shocks breaks this symmetry because the arrival of new idiosyncratic shocks is independent of the firm’s state. As a result, a fraction of firms does not adjust regardless of their initial employment, a feature reminiscent of the Calvo model.\(^{49}\)

\(^{48}\)Numerically, Midrigan (2011) finds that an AR(1) process for the log price level is very accurate. This is precisely the implication of the analytical approximation in (24). Moreover, Midrigan’s simulated degree of persistence (see his Table III) implies the same kind of hump-shaped pattern we see in our Figure 5.

\(^{49}\)It is not the discreteness of the productivity process per se that matters: we find numerically that the neutrality result of Proposition 2 obtains even if the distribution of \( x \) is discretized. Rather, what is special about the Poisson process is that the distribution of \( x \) has a discrete mass point at \( x = x_{-1} \), regardless of the firm’s past employment or productivity. This induces an (approximately) exogenous component to the adjustment decision that weakens the selection effect.
5 Summary and Discussion

Our analysis of the aggregate implications of a canonical model of fixed employment adjustment costs has established a stark neutrality result. In general, the dynamics of aggregate employment in the presence of an adjustment friction can be inferred simply and intuitively by characterizing the evolution of the distribution of employment across firms. We show that, to a first-order approximation, aggregate employment dynamics coincide with their frictionless counterpart, even in the absence of equilibrium adjustment of market prices. This result arises from a form of symmetry in the dynamics of the firm-size distribution that emerges as the adjustment cost becomes small. In that neighborhood, we show that the probability that a firm adjusts to a given employment level is approximately offset by the probability that a firm adjusts away from that level, leaving the path of the firm-size distribution almost unimpaired.

Thus, our analysis provides an analytical foundation to recent quantitative research on the macroeconomic effects of discrete adjustment costs in a general framework. It provides a precise formal justification for the approximate neutrality noted in numerical simulations by Golosov and Lucas (2007) in the context of a related menu cost model. Similarly, our own quantitative analysis of a model of employment adjustment calibrated to leading estimates of adjustment costs imply aggregate dynamics that are close to frictionless outcomes, also in line with our approximate neutrality result.

Our analysis also offers a novel perspective on the circumstances in which aggregate dynamics can be expected to deviate from their frictionless counterparts. A unifying theme in our findings is the important role of symmetry in unwinding the aggregate effects of lumpy adjustment. It follows that deviations from frictionless dynamics can be traced to violations of this symmetry. We show that an important example of the latter is recent research that has invoked compound Poisson processes of idiosyncratic shocks in which only a fraction $1 - \lambda$ of firms receives a shock each period (Gertler and Leahy, 2008; Midrigan, 2011). Our approximations provide a novel perspective on this result: to a first order approximation, we demonstrate that implied aggregate dynamics in this case are isomorphic to Calvo adjustment with exogenous adjustment parameter $1 - \lambda$.

These results highlight a number of interesting avenues for future research. First, since the magnitude of adjustment costs and idiosyncratic risk play a role in the model’s aggregate dynamics, it remains important for empirical work to focus on obtaining robust estimates of these two critical parameters. Second, we join the influential recent work of Gertler and Leahy and Midrigan in emphasizing the role of the form of idiosyncratic productivity shocks.
Given its theoretical importance, future empirical work that estimates the distribution of idiosyncratic shocks will be of particular value.

To the extent that estimates of these parameters line up with the approximate aggregate neutrality we identify, it is worthwhile to consider other adjustment frictions that simultaneously can account for lumpy microeconomic adjustment and persistent aggregate dynamics. For instance, both fixed and kinked (proportional) adjustment costs induce inaction at the microeconomic level, but may have very different implications for aggregate employment dynamics. In addition, there may be additional frictions, or technological constraints, to which the firm is subject that interact with adjustment costs. For instance, Bachmann, Caballero, and Engel (2013) consider a model in which there are “core components” to the capital stock whose depreciation must be replaced in order for the plant to operate. They argue that this feature can amplify the effects of a fixed cost of capital adjustment on the aggregate dynamics of investment.

Our framework would suggest that, to the extent these other frictions alter the dynamics, they must disrupt the symmetry of the adjustment policy. And indeed, using plant-level data on employment and investment, the analysis of Caballero, Engel, and Haltiwanger (1995, 1997) does suggest that asymmetries are important empirically. The question of what lies behind this asymmetry—and what it implies for the aggregate dynamics—is thus an important topic for future research.

6 References


Figure 1. S$S$ labor demand policy in baseline parameterization (bold line, illustrated for $n_{-1} = 25$)

Figure 2. Steady-state distribution of employment in baseline parameterization
Figure 3. Dynamic response in baseline parameterization

A. Impulse response to a one-percent innovation to $p$

B. Aggregate reductions in flows
Figure 4. Sensitivity analysis

A. Larger adjustment cost, $C/E(y) = 0.16$

B. Lower idiosyncratic dispersion, $\sigma_x = 0.2$

C. Match frequency and size of adjustments

D. Varying idiosyncratic persistence, $\rho_x$

E. Stochastic adjustment costs

F. Size-dependent adjustment costs
Compound Poisson idiosyncratic shocks

Table 1. Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Returns to scale</td>
<td>0.64</td>
<td>Cooper, Haltiwanger and Willis (2005)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
<td>Quarterly real interest rate = 1%</td>
</tr>
<tr>
<td>$C/E(y)$</td>
<td>Adj. cost / Avg. revenue</td>
<td>0.08</td>
<td>Cooper, Haltiwanger and Willis (2005); Bloom (2009)</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Persistence of $x$</td>
<td>0.70</td>
<td>Cooper, Haltiwanger and Willis (2005); Foster, Haltiwanger and Syverson (2008)</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Std. dev. of innovation to $x$</td>
<td>0.35</td>
<td>Cooper, Haltiwanger and Willis (2005, 2007)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Persistence of $p$</td>
<td>0.95</td>
<td>Autocorrelation of detrended log $N$</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Std. dev. of innovation to $p$</td>
<td>0.015</td>
<td>Std. dev. of detrended log $N$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Worker attrition rate</td>
<td>0.06</td>
<td>Quarterly quit rate (JOLTS)</td>
</tr>
</tbody>
</table>
A Proofs of Lemmas and Propositions

In this appendix, we prove the results in the main text in the more general case with worker attrition. If an exogenous fraction \( \delta \) of a firm’s workforce separates each period, the expected present discounted value of a firm’s profits is given by:

\[
\Pi (\tilde{n}_{-1}, x; \Omega) \equiv \max_n \left\{ px F(n) - wn - C 1 [n \neq \tilde{n}_{-1}] + \beta \mathbb{E} [\Pi (\tilde{n}, x'; \Omega') | x, \Omega] \right\},
\]

where \( \tilde{n}_{-1} \equiv (1 - \delta) n_{-1} \) denotes employment carried into the period. Thus, the “reset” function \( X(n) \) satisfies the first-order condition

\[
pX (n) F_n (n) - w + \beta (1 - \delta) \mathbb{E} [\Pi_1 (\tilde{n}, x'; \Omega') | x = X(n), \Omega] \equiv 0,
\]

and the adjustment triggers satisfy the value-matching conditions

\[
\Pi^\Delta (L (\tilde{n}_{-1}; \Omega); \Omega) - C = \Pi^0 (\tilde{n}_{-1}, L (\tilde{n}_{-1}; \Omega); \Omega), \quad \text{and}
\]

\[
\Pi^\Delta (U (\tilde{n}_{-1}; \Omega); \Omega) - C = \Pi^0 (\tilde{n}_{-1}, U (\tilde{n}_{-1}; \Omega); \Omega).
\]

**Proof of Proposition 1.** Consider the inflow into the mass \( H(m) \)---i.e. the mass of firms that cuts employment from above \( m \) to below \( m \). To derive this flow, first fix a level of lagged employment \( n_{-1} \), and denote the distribution of productivity conditional on lagged employment as \( \mathcal{G} (\xi | \nu) \equiv \Pr [x \leq \xi | n_{-1} = \nu] \). Figure 1 identifies four potential sets of inflows:

1) If \( m < X^{-1} L (\tilde{n}_{-1}) \), so that \( n_{-1} > \frac{L^{-1} X(m)}{1 - \delta} \), the probability of reducing employment below \( m \) will be \( \mathcal{G} [X(m) | n_{-1}] \).

2) If \( m \in [X^{-1} L (\tilde{n}_{-1}), \tilde{n}_{-1}] \), so that \( n_{-1} \in \left[ \frac{m}{1 - \delta}, \frac{L^{-1} X(m)}{1 - \delta} \right] \), the probability of reducing employment below \( m \) will be \( \mathcal{G} [L (\tilde{n}_{-1}) | n_{-1}] \).

3a) If \( n_{-1} < X^{-1} U (\tilde{n}_{-1}) \), and \( m \in [\tilde{n}_{-1}, n_{-1}] \), so that \( n_{-1} \in [m, \frac{m}{1 - \delta}] \), the probability of reducing employment below \( m \) will be \( \mathcal{G} [U (\tilde{n}_{-1}) | n_{-1}] \).

3b) If \( n_{-1} > X^{-1} U (\tilde{n}_{-1}) \), and \( m \in [\tilde{n}_{-1}, X^{-1} U (\tilde{n}_{-1})] \), so that \( n_{-1} \in \left[ \frac{X^{-1} X(m)}{1 - \delta}, \frac{m}{1 - \delta} \right] \), the probability of reducing employment below \( m \) will be \( \mathcal{G} [U (\tilde{n}_{-1}) | n_{-1}] \).

4b) If \( n_{-1} > X^{-1} U (\tilde{n}_{-1}) \), and \( m \in [X^{-1} U (\tilde{n}_{-1}), n_{-1}] \), so that \( n_{-1} \in \left[ m, \frac{X^{-1} X(m)}{1 - \delta} \right] \), the probability of reducing employment below \( m \) will be \( \mathcal{G} [X(m) | n_{-1}] \).

It follows that the inflow is given by

\[
\text{Inflow to } H(m) = \int_{\frac{L^{-1} X(m)}{1 - \delta}}^{\infty} \mathcal{G} [X(m) | n_{-1}] dH_{-1} (n_{-1}) + \int_{\frac{m}{1 - \delta}}^{\frac{L^{-1} X(m)}{1 - \delta}} \mathcal{G} [L (\tilde{n}_{-1}) | n_{-1}] dH_{-1} (n_{-1})
\]

\[
+ \int_{\max \left\{ m, \frac{X^{-1} X(m)}{1 - \delta} \right\}}^{\frac{m}{1 - \delta}} \mathcal{G} [U (\tilde{n}_{-1}) | n_{-1}] dH_{-1} (n_{-1})
\]

\[
+ \int_{m}^{\max \left\{ m, \frac{X^{-1} X(m)}{1 - \delta} \right\}} \mathcal{G} [X(m) | n_{-1}] dH_{-1} (n_{-1}),
\]

(28)
By the same logic, the outflow from the mass \( H(m) \) is the mass of firms that raises employment from below \( m \). Figure 1 identifies two potential sets of outflows:

4a) If \( n_{-1} < X^{-1}U(\tilde{n}_{-1}) \), and \( m \in [n_{-1}, X^{-1}U(\tilde{n}_{-1})] \), so that \( n_{-1} \in \left[ \frac{U^{-1}X(m)}{1-\delta}, m \right] \), the probability of increasing employment above \( m \) will be \( 1 - G[U(\tilde{n}_{-1}) | n_{-1}] \).

5a) If \( n_{-1} < X^{-1}U(\tilde{n}_{-1}) \), and \( m > X^{-1}U(\tilde{n}_{-1}) \), so that \( n_{-1} < \frac{U^{-1}X(m)}{1-\delta} \), the probability of increasing employment above \( m \) will be \( 1 - G[X(m) | n_{-1}] \).

5b) If \( n_{-1} > X^{-1}U(\tilde{n}_{-1}) \), and \( m > n_{-1} \), so that \( n_{-1} < m \), the probability of increasing employment above \( m \) will be \( 1 - G[X(m) | n_{-1}] \).

It follows that the outflow is given by

\[
\text{Outflow from } H(m) = \int_{\min\left\{ m, \frac{U^{-1}X(m)}{1-\delta} \right\}}^{m} (1 - G[U(\tilde{n}_{-1}) | n_{-1}]) \, dH_{-1}(n_{-1}) \\
+ \int_{0}^{\min\left\{ m, \frac{U^{-1}X(m)}{1-\delta} \right\}} (1 - G[X(m) | n_{-1}]) \, dH_{-1}(n_{-1}).
\] (29)

The mass of firms with employment below some level \( n \) this period is equal to the mass below \( n \) in the previous period plus inflows into the mass less outflows from the mass. Thus, using equations (28) and (29) we can express the evolution of the distribution function \( H(n) \) as

\[
\Delta H(m) = G[X(m)] - H_{-1}(m) - \int_{\frac{U^{-1}X(m)}{1-\delta}}^{\frac{L^{-1}X(m)}{1-\delta}} G[X(m) | n_{-1}] \, dH_{-1}(n_{-1})
\]

\[
+ \int_{\frac{U^{-1}X(m)}{1-\delta}}^{\frac{L^{-1}X(m)}{1-\delta}} G[L(\tilde{n}_{-1}) | n_{-1}] \, dH_{-1}(n_{-1}) + \int_{\frac{U^{-1}X(m)}{1-\delta}}^{m} G[U(\tilde{n}_{-1}) | n_{-1}] \, dH_{-1}(n_{-1}).
\] (30)

Differentiating, denoting the frictionless density of employment as \( h^*(m) \equiv g[X(m)]X'(m) \), using Bayes’ rule to write the distribution of lagged employment conditional on current productivity as \( \mathcal{H}(\nu | \xi) \equiv \text{Pr}[n_{-1} \leq \nu | x = \xi] = \int_{0}^{\nu} \frac{1}{g(\xi)} \, dH_{-1}(\tilde{\nu}) \), and defining \( \tilde{m} \equiv (1 - \delta)m \) yields the stated result,

\[
\Delta h(\tilde{m}) = -[h_{-1}(\tilde{m}) - \tilde{h}(\tilde{m})] + \frac{1}{1-\delta} (G[U(\tilde{m}) | m] - G[L(\tilde{m}) | m]) [h_{-1}(m) - \tilde{h}(m)],
\] (31)

where the steady-state density satisfies the recursion

\[
\tilde{h}(\tilde{m}) = \left( 1 - \mathcal{H} \left[ \frac{L^{-1}X(\tilde{m})}{1-\delta} \big| X(\tilde{m}) \right] + \mathcal{H} \left[ \frac{U^{-1}X(\tilde{m})}{1-\delta} \big| X(\tilde{m}) \right] \right) h^*(\tilde{m})
\]

\[
+ \frac{1}{1-\delta} (G[U(\tilde{m}) | m] - G[L(\tilde{m}) | m]) \tilde{h}(m).
\] (32)

Setting \( \delta = 0 \) yields the result stated in Proposition 1.
Proof of Lemma 1. The proof holds for any given aggregate state $\Omega$, and so for transparency we suppress dependence on $\Omega$ in what follows. Recall that the adjustment triggers satisfy the value matching condition, $\Delta (\tilde{n}, x') \equiv \Pi^0 (x') - \Pi^0 (\tilde{n}, x') = C$. In the presence of $C \approx 0$, we may restrict our focus to a second-order approximation to $\Delta (\tilde{n}, x')$ around $x' = X(\tilde{n})$:
\[
\Delta (\tilde{n}_-, x) \approx \Delta (\tilde{n}_-, X(\tilde{n}_-)) + \Delta_x (\tilde{n}_-, X(\tilde{n}_-)) (x - X (\tilde{n}_-)) + \frac{1}{2} \Delta_{xx} (\tilde{n}_-, X(\tilde{n}_-)) (x - X (\tilde{n}_-))^2.
\] (33)

The first and second terms on the right side are zero by optimality. Setting $\Delta (\tilde{n}_-, x) = C$, it follows that the triggers are as stated in (11).

The inverse triggers may be derived symmetrically by approximating $\Delta (\tilde{n}_-, x)$ around $\tilde{n}_- = X^{-1}(x)$:
\[
\Delta (\tilde{n}_-, x) \approx \Delta (X^{-1}(x), x) + \Delta_1 (X^{-1}(x), x) (\tilde{n}_- - X^{-1}(x)) + \frac{1}{2} \Delta_{11} (X^{-1}(x), x) (\tilde{n}_- - X^{-1}(x))^2.
\] (34)

Again, optimality implies the first two terms in the expansion are zero. Setting $\Delta (\tilde{n}_-, x) = C$ yields the stated inverse triggers in (12).

To complete the proof, define the firm’s objective function, gross of the adjustment cost, by $\Theta (n, x) \equiv px F (n) - wn + \beta \int \Pi (\tilde{n}, x') dG (x'|x)$. Note that $\Delta (\tilde{n}_-, x) = \Theta (X^{-1}(x), x) - \Theta (\tilde{n}_-, x)$. It follows that $\Delta_{11} (\tilde{n}_-, x) = -\Theta_{nn} (\tilde{n}_-, x)$, and that
\[
\Delta_{xx} (\tilde{n}_-, x) = \Theta_{n} (X^{-1}(x), x) \frac{\partial^2 X^{-1}(x)}{\partial x^2} + \Theta_{nn} (X^{-1}(x), x) \left[ \frac{\partial X^{-1}(x)}{\partial x} \right]^2 + 2 \Theta_{nx} (X^{-1}(x), x) \frac{\partial X^{-1}(x)}{\partial x} + \Theta_{xx} (X^{-1}(x), x) - \Theta_{xx} (\tilde{n}_-, x).
\] (35)

By optimality, we know that $\Theta_{n} (X^{-1}(x), x) \equiv 0$. It follows that $\Theta_{nn} (X^{-1}(x), x) \frac{\partial X^{-1}(x)}{\partial x} + \Theta_{nx} (X^{-1}(x), x) = 0$. Thus, we can rewrite (35) as
\[
\Delta_{xx} (\tilde{n}_-, x) = -\Theta_{nn} (X^{-1}(x), x) \left[ \frac{\partial X^{-1}(x)}{\partial x} \right]^2 + \Theta_{xx} (X^{-1}(x), x) - \Theta_{xx} (\tilde{n}_-, x).
\] (36)

Recalling from above that $\Delta_{11} (\tilde{n}_-, X(\tilde{n}_-)) = -\Theta_{nn} (\tilde{n}_-, X(\tilde{n}_-))$, noting that $\frac{\partial X^{-1}(x)}{\partial x} = \left[ X' (X^{-1}(x)) \right]^{-1}$, and evaluating at $x = X (\tilde{n}_-)$ yields
\[
\Delta_{xx} (\tilde{n}_-, X(\tilde{n}_-)) = \Delta_{11} (\tilde{n}_-, X(\tilde{n}_-)) \left[ \frac{1}{X'(\tilde{n}_-)} \right]^2,
\] (37)

which implies that $\gamma (\tilde{n}) = X'(\tilde{n}) \tilde{\gamma} (X(\tilde{n}))$, as required. \[\square\]
Proof of Lemma 2. If the firm adjusts next period, it will earn \( \Pi^t (x', \Omega') - C \), for a given \( x' \). But the firm can do better by not adjusting if \( x' \in [L(n, \Omega'), U(n, \Omega')] \). In this region, the fixed cost, \( C \), outweighs the net value of adjusting, \( \Delta(n, x', \Omega') \equiv \Pi^t(n, \Omega') - \Pi^0(n, x', \Omega') \). The expected value of the firm can thus be calculated by augmenting the return from always adjusting by the value of inaction in this region:

\[
\mathbb{E}_{\Omega'} \left[ \int \Pi(n, x', \Omega') \ dG(x'|x) \right]
= \mathbb{E}_{\Omega'} \left[ \int [\Pi^t(x', \Omega') - C] \ dG(x'|x) + \int_{L(n, \Omega')}^{U(n, \Omega')} [C - \Delta(n, x', \Omega')] \ dG(x'|x) \right],
\]

(38)

where the expectation outside the brackets is taken with respect to the distribution of the aggregate state \( \Omega' \). The current level of employment \( n \) enters only through the second term inside the brackets, which in turn can be written as

\[
(G[U(n, \Omega')|x] - G[L(n, \Omega')|x]) \int_{L(n, \Omega')}^{U(n, \Omega')} \frac{C - \Delta(n, x', \Omega')}{G[U(n, \Omega')|x] - G[L(n, \Omega')|x]} \ dG(x'|x).
\]

(39)

Optimality ensures that \( \Delta(n, x', \Omega') \in [0, C] \) for all \( x' \). Hence, \( C - \Delta(n, x', \Omega') \) is of order \( C \). Since the expectation of an order-\( C \) random variable is order \( C \), the integral is order \( C \). To assess the leading term of the product in (39), take a second-order approximation to \( G(x'|x) \) around \( x' = X(n) \) and then evaluate this at \( x' = U(n, \Omega') \) and, separately, at \( x' = L(n, \Omega') \). Taking the difference of these two, and using Lemma 1 to substitute for the triggers, we have,

\[
G[U(n, \Omega')|x] - G[L(n, \Omega')|x] \approx 2g[X(n, \Omega')|x] \gamma(n, \Omega') \sqrt{C}.
\]

(40)

Substituting this into (39), it follows that (39) is of order \( C^{3/2} \). Hence, the current level of employment \( n \) affects the future value of the firm only via higher-order terms in \( C \). ■

Lemma 3 The evolution of the density of idiosyncratic productivity conditional on lagged employment satisfies the dynamic equation

\[
G'(x|n_{-1}) = \pi_{-1} \frac{\int_{L(n_{-1})}^{U(n_{-1})} g(x|x_{-1}) G'_{-1}(x_{-1}|n_{-1}) \ dx_{-1}}{G_{-1}[U(n_{-1})|n_{-1}] - G_{-1}[L(n_{-1})|n_{-1}]} + (1 - \pi_{-1}) g(x|X(n_{-1})).
\]

(41)

where \( \pi_{-1} \equiv \text{Pr}[n_{-1} = \tilde{n}_{-1}] \) is the probability of not adjusting last period. If \( g \) is analytic, the law of motion preserves analyticity of \( G' \).
Proof of Lemma 3. First note that we may write $\mathcal{G}(x|n_{-1}) = \int G(x|x_{-1}) \, d\mathcal{G}(x_{-1}|n_{-1})$, where

$$
\mathcal{G}(\xi|n_{-1}) \equiv \Pr[x_{-1} \leq \xi|n_{-1}]
= \Pr[x_{-1} \leq \xi|n_{-1}, n_{-1} = \tilde{n}_{-2}] \Pr[n_{-1} = \tilde{n}_{-2}]
+ \Pr[x_{-1} \leq \xi|n_{-1}, n_{-1} \neq \tilde{n}_{-2}] \Pr[n_{-1} \neq \tilde{n}_{-2}].
$$

(42)

In the event that the firm adjusted last period, $n_{-1} \neq \tilde{n}_{-2}$, we know that the firm would have adjusted so that $x_{-1} = X(n_{-1})$. Thus,

$$
\Pr[x_{-1} \leq \xi|n_{-1}, n_{-1} \neq \tilde{n}_{-2}] = 1 \left[ \xi \geq X(n_{-1}) \right].
$$

(43)

In the case in which the firm did not adjust last period, we know $n_{-2}$. That information alone implies that $x_{-1}$ will be distributed according to the c.d.f of $x_{-1}|n_{-2}$, which we denote by $\mathcal{G}_{-1}$, the lagged counterpart of $\mathcal{G}$. In addition, however, we also know that $n_{-1} = \tilde{n}_{-2}$. This implies that $x_{-1} \in [L(n_{-1}), U(n_{-1})]$, but is otherwise uninformative on the distribution of $x_{-1}$. Thus,

$$
\Pr[x_{-1} \leq \xi|n_{-1}, n_{-1} = \tilde{n}_{-2}] = \frac{\mathcal{G}_{-1}(\xi|\frac{n_{-1}}{1-\delta}) - \mathcal{G}_{-1}[L(n_{-1})|\frac{n_{-1}}{1-\delta}]}{\mathcal{G}_{-1}[U(n_{-1})|\frac{n_{-1}}{1-\delta}] - \mathcal{G}_{-1}[L(n_{-1})|\frac{n_{-1}}{1-\delta}]].
$$

(44)

Defining

$$
\pi_{-1} \equiv \Pr[n_{-1} = \tilde{n}_{-2}] = \int (\mathcal{G}_{-1}[U(\tilde{n}_{-2})|n_{-2}] - \mathcal{G}_{-1}[L(\tilde{n}_{-2})|n_{-2}]) \, dH_{-2}(n_{-2}),
$$

(45)

we can therefore write

$$
\mathcal{G}(x_{-1}|n_{-1}) = \pi_{-1} \frac{\mathcal{G}_{-1}(x_{-1}|\frac{n_{-1}}{1-\delta}) - \mathcal{G}_{-1}[L(n_{-1})|\frac{n_{-1}}{1-\delta}]}{\mathcal{G}_{-1}[U(n_{-1})|\frac{n_{-1}}{1-\delta}] - \mathcal{G}_{-1}[L(n_{-1})|\frac{n_{-1}}{1-\delta}]]} + (1 - \pi_{-1}) 1 \left[ x_{-1} \geq X(n_{-1}) \right].
$$

(46)

Substituting into the definition of $\mathcal{G}(x|n_{-1})$ yields its dynamic update equation,

$$
\mathcal{G}(x|n_{-1}) = \pi_{-1} \frac{\int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})} G(x|x_{-1}) \mathcal{G}_{-1}(x_{-1}|\frac{n_{-1}}{1-\delta}) \, dx_{-1}}{\mathcal{G}_{-1}[U(n_{-1})|\frac{n_{-1}}{1-\delta}] - \mathcal{G}_{-1}[L(n_{-1})|\frac{n_{-1}}{1-\delta}]]} + (1 - \pi_{-1}) G(x|X(n_{-1})).
$$

(47)

That the latter preserves analyticity of $\mathcal{G}$ follows from analyticity of $G$, the fact that sums, products and integrals of analytic functions are themselves analytic, and that quotients of analytic functions with a non-zero denominator are also analytic. ■
Proof of Proposition 2. We seek to prove that \( \partial [\Delta h(\tilde{n})]/\partial C|_{C=0} = 0 \). To that end, recall the law of motion for the density of employment \( h(n) \), which we rewrite here as

\[
\Delta h(\tilde{n}) = \left(1 - \mathcal{H} \left[ \frac{L^{-1}X(\tilde{n})}{1 - \delta} \right] - X(\tilde{n}) \right) + \mathcal{H} \left[ \frac{U^{-1}X(\tilde{n})}{1 - \delta} \right] h^*(\tilde{n})
+ \frac{1}{1 - \delta} \left( \mathcal{G} [U(\tilde{n})|n] - \mathcal{G} [L(\tilde{n})|n] \right) h_{-1}(n) - h_{-1}(\tilde{n}).
\] (48)

Taking the previous period’s density of employment \( h_{-1}(\cdot) \) as predetermined, we conjecture that \( \partial \mathcal{G}/\partial C|_{C=0} = 0 \), and confirm that this is so later in the proof. Note from the proof of Proposition 1 that, since \( \mathcal{H}(\nu|\xi) = \int_{0}^{\nu} \frac{g'(\xi|\nu)}{g(\xi)} dH_{-1}(\nu) \), it follows that \( \partial \mathcal{H}/\partial C|_{C=0} = 0 \) under the conjecture. In addition, from Lemma 2 we know that \( \partial X/\partial C|_{C=0} = 0 \). Given the conjecture we can therefore write the derivative of the law of motion (48) with respect to the adjustment cost in the neighborhood of \( C = 0 \) as

\[
\frac{\partial [\Delta h(\tilde{n})]}{\partial C} \approx \left\{ \mathcal{H}' \left[ \frac{L^{-1}X(\tilde{n})}{1 - \delta} \right] - \mathcal{H}' \left[ \frac{U^{-1}X(\tilde{n})}{1 - \delta} \right] h^*(\tilde{n}) \right\} \frac{1}{1 - \delta} \\
+ \left\{ \mathcal{G}' [U(\tilde{n})|n] \frac{\partial U(\tilde{n})}{\partial C} - \mathcal{G}' [L(\tilde{n})|n] \frac{\partial L(\tilde{n})}{\partial C} \right\} h_{-1}(n) \right\} \frac{1}{1 - \delta}.
\] (49)

Using Lemma 1 to approximate \( \frac{L^{-1}X(\tilde{n})}{1 - \delta}, \frac{U^{-1}X(\tilde{n})}{1 - \delta}, U(\tilde{n}), L(\tilde{n}) \) and their derivatives in the neighborhood of \( C = 0 \), and recalling that \( \gamma(\tilde{n}) = X'(\tilde{n}) \gamma(X(\tilde{n})) \), yields:

\[
\frac{\partial [\Delta h(\tilde{n})]}{\partial C} \approx \left\{ \mathcal{H}' \left[ \frac{\gamma(X(\tilde{n}))}{1 - \delta} \sqrt{C} \right] X(\tilde{n}) \right\} \frac{\gamma(X(\tilde{n})) h^*(\tilde{n})}{1 - \delta} \\
+ \left\{ \mathcal{G}' \left[ \gamma(\tilde{n}) \sqrt{C} \right] + \mathcal{G}' \left[ X(\tilde{n}) - \gamma(\tilde{n}) \sqrt{C} \right] \right\} X'(\tilde{n}) \frac{\gamma(X(\tilde{n})) h_{-1}(n)}{2 \sqrt{C}}.
\] (50)

We seek to infer the limiting behavior of the latter as \( C \to 0 \). Expanding the terms in braces and canceling yields:

\[
\frac{\partial [\Delta h(\tilde{n})]}{\partial C} \approx \left\{ 2 \mathcal{H}' [n] X(\tilde{n}) + \mathcal{H}''' [n] X(\tilde{n}) \left( \frac{\gamma(X(\tilde{n}))}{1 - \delta} \right)^{2} C + ... \right\} \frac{\gamma(X(\tilde{n})) h^*(\tilde{n})}{1 - \delta} \\
+ \left\{ 2 \mathcal{G}' [X(\tilde{n})|n] + \mathcal{G}''' [X(\tilde{n})|n] \gamma(\tilde{n})^{2} C + ... \right\} X'(\tilde{n}) \frac{\gamma(X(\tilde{n})) h_{-1}(n)}{2 \sqrt{C}}.
\] (51)

The limiting behavior of the latter will be dominated by the leading terms in braces, that is

\[
\frac{\partial [\Delta h(\tilde{n})]}{\partial C} \approx \left\{ - \mathcal{H}' [n] X(\tilde{n}) h^*(\tilde{n}) + \mathcal{G}' [X(\tilde{n})|n] X'(\tilde{n}) h_{-1}(n) \right\} \frac{\gamma(X(\tilde{n}))}{(1 - \delta) \sqrt{C}} + O \left( \sqrt{C} \right).
\] (52)

Finally, recall that the frictionless density of employment \( h^*(\tilde{n}) = X'(\tilde{n}) \gamma[X(\tilde{n})] \), and that, from the definitions of \( \mathcal{G}'(\xi|\nu) \) and \( \mathcal{H}'(\nu|\xi) \), the above conjecture implies that \( \mathcal{G}' [X(\tilde{n})|n] \approx g \left[ X(\tilde{n}) | X(n) \right] \), and \( \mathcal{H}' [n] X(\tilde{n}) \approx g \left[ X(\tilde{n}) | X(n) \right] \frac{h_{-1}(n)}{g(X(\tilde{n}))} \) in the neighborhood of \( C = 0 \). Substitution confirms that \( \partial [\Delta h(n)]/\partial C|_{C=0} = 0 \), as required.
It remains to verify that \( \partial G / \partial C |_{C=0} = 0 \). First, rewrite the law of motion for \( G \) as

\[
G' (x|n_{-1}) = \pi_{-1} \int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})} \left[ g(x|X(n_{-1})) - g(x|X(n_{-1})) \right] G'_{-1} (x_{-1}|n_{-1}^{\frac{1}{1-\delta}}) \, dx_{-1} + g(x|X(n_{-1})) ,
\]

and recall the probability of not adjusting last period \( \pi_{-1} \) in equation (45). Using Lemma 1 and the conjecture, it follows that \( \pi_{-1} = O \left( \sqrt{C} \right) \) and \( G_{-1} \left[ U (\tilde{n}_{-1}) \left| n_{-1}^{\frac{1}{1-\delta}} \right. \right] - G_{-1} \left[ L (\tilde{n}_{-1}) \left| n_{-1}^{\frac{1}{1-\delta}} \right. \right] = O \left( \sqrt{C} \right) \). Thus, the limiting behavior of \( G' (x|n_{-1}) \) as \( C \to 0 \) is determined by the limiting behavior of the remaining term, \( A \equiv \int_{L(\tilde{n}_{-1})}^{U(\tilde{n}_{-1})} \left[ g(x|X(n_{-1})) - g(x|X(n_{-1})) \right] G'_{-1} (x_{-1}|n_{-1}^{\frac{1}{1-\delta}}) \, dx_{-1} \).

Under the conjecture,

\[
\frac{\partial A}{\partial C} \approx \left[ g(x|U(\tilde{n}_{-1})) - g(x|X(n_{-1})) \right] G'_{-1} \left( U (\tilde{n}_{-1}) \left| n_{-1}^{\frac{1}{1-\delta}} \right. \right) \frac{\partial U(\tilde{n}_{-1})}{\partial C}
- \left[ g(x|L(\tilde{n}_{-1})) - g(x|X(n_{-1})) \right] G'_{-1} \left( L (\tilde{n}_{-1}) \left| n_{-1}^{\frac{1}{1-\delta}} \right. \right) \frac{\partial L(\tilde{n}_{-1})}{\partial C} .
\]

From Lemmas 1 and 3, in the neighborhood of \( C = 0 \) we can write the latter as

\[
\frac{\partial A}{\partial C} \approx g_2 (x|X(\tilde{n}_{-1})) g' \left( X(\tilde{n}_{-1}) | X \left( n_{-1}^{\frac{1}{1-\delta}} \right) \right) \gamma (\tilde{n}_{-1}) \sqrt{C} \to 0 \; \text{as} \; C \to 0.
\]

It follows that \( \partial G / \partial C |_{C=0} = 0 \), as required. ■

**Lemma 4** Consider the case in which \( x \) is i.i.d. If the adjustment cost \( C (x) \equiv c R (x) \), \( R (x) \) is twice-differentiable, increasing, and convex, and orders greater than \( c \) are negligible, then approximate aggregate neutrality continues to hold.

**Proof of Lemma 4.** The proof proceeds in four stages:

1. Adjustment triggers, \( \theta (n) \in \{ L (n), U (n) \} \). A second-order approximation to the value-matching condition \( \Delta (n_{-1}, x) - c R (x) = 0 \) around \( x = X(n_{-1}) \) yields:

\[
\frac{1}{2} \left[ \Delta_{xx} (n_{-1}, X(n_{-1})) - c R'' (X(n_{-1})) \right] (x - X(n_{-1}))^2
- c R' (X(n_{-1})) (x - X(n_{-1})) - c R (X(n_{-1})) \approx 0 ,
\]

where we have used the fact that, by optimality, \( \Delta (n_{-1}, X(n_{-1})) = \Delta_x (n_{-1}, X(n_{-1})) = 0 \). Solving this quadratic implies

\[
\theta (n) \approx X (n) + \psi (c, n) \pm \sqrt{\frac{2 R (X(n))}{R'(X(n))}} \psi (c, n) \sqrt{1 + \frac{R' (X(n))}{2 R (X(n))} \psi (c, n)} ,
\]

where

\[
\psi (c, n) \equiv \frac{c R' (X(n))}{\Delta_{xx} (n, X(n)) - c R'' (X(n))} \approx \frac{c R' (X(n))}{\Delta_{xx} (n, X(n))} + O (c^2) .
\]
cSince $\sqrt{O(c)} = O(\sqrt{c})$, $\sqrt{1 + O(c)} = 1 + O(c)$, and orders higher than $c$ are taken to be negligible, it follows that
\[
\theta(n) \approx X(n) + \eta(n) c \pm \gamma(n) \sqrt{c},
\]
where $\eta(n) \equiv R'(X(n))/\Delta_{xx}(n, X(n))$, and $\gamma(n) \equiv \sqrt{2R(X(n))/\Delta_{xx}(n, X(n))}$. The presence of the term $\eta(n) c$ distinguishes the case of size-dependent adjustment costs from the lump-sum counterpart.

2. Inverse adjustment triggers, $\theta^{-1}(x) \in \{U^{-1}(x), L^{-1}(x)\}$. Since the adjustment cost is independent of $n$, as before, a second order approximation to the value-matching condition $\Delta(n, x) - cR(x) = 0$ around $n = X^{-1}(x)$ yields:
\[
\theta^{-1}(x) \approx X^{-1}(x) \pm \tilde{\gamma}(x) \sqrt{c},
\]
where we have noted that $\Delta(X^{-1}(x), x) = \Delta_n(X^{-1}(x), x) = 0$ by optimality, and $\tilde{\gamma}(x) \equiv \sqrt{2R(x)/\Delta_{nn}(X^{-1}(x), x)}$. As in Lemma 1, $\gamma(n) = X'(n) \tilde{\gamma}(X(n))$.

3. Approximate optimality of myopia. Lemma 2 continues to hold via a trivial extension of its proof.

4. Approximate neutrality. Since Lemma 2 obtains, the mandated density coincides with its frictionless counterpart, $h^*(n)$. As before, the above approximations for the inverse triggers imply the reduction in the inflow to $n$ relative to the frictionless case can be approximated by $h^*(n) \left(H_{-1}[L^{-1}X(n)] - H_{-1}[U^{-1}X(n)]\right) \approx 2h_{-1}(n) h^*(n) \tilde{\gamma}(X(n)) \sqrt{c}$. The reduction in the outflow from $n$ relative to the frictionless case $h_{-1}(n) \left(G[U(n)] - G[L(n)]\right)$ is slightly more complicated, since the above approximations to the triggers differ from the lump-sum case. But, the upshot is the same when orders greater than $c$ are negligible. In particular, the reduction in outflows is approximated by:
\[
\begin{align*}
&h_{-1}(n) \left[ G[X(n)] + g[X(n)] (\eta(n) c + \gamma(n) \sqrt{c}) + \frac{1}{2} g'[X(n)] (\eta(n) c + \gamma(n) \sqrt{c})^2 \right. \\
&\left. - G[X(n)] - g[X(n)] (\eta(n) c - \gamma(n) \sqrt{c}) - \frac{1}{2} g'[X(n)] (\eta(n) c - \gamma(n) \sqrt{c})^2 \right] \\
&\approx 2h_{-1}(n) g[X(n)] \gamma(n) \sqrt{c} = 2h_{-1}(n) h^*(n) \tilde{\gamma}(X(n)) \sqrt{c}.
\end{align*}
\]
Thus, the reduction in inflows is approximately equal to the reduction in outflows, and approximate neutrality obtains.
B Quantitative Analysis: Wage Adjustment

In this appendix, we extend the quantitative results reported in the main text to the case with equilibrium wage adjustment. We specify the supply side of the market by introducing an upward-sloped aggregate labor supply schedule of the loglinear form\(^{50}\)

\[
N^* (w) = \psi w^\eta.
\]  

(62)

A firm must now forecast future wages in order to solve for labor demand. This is a challenging problem because the future wage is jointly determined with future aggregate employment, and the latter derives from the (future) distribution of employment across firms \(h(n)\)—an infinite-dimensional object. To circumvent the dimensionality of the problem, we assume firms employ Krusell and Smith’s (1998) bounded rationality algorithm. Specifically, firms forecast aggregate employment \(N'\) using only the first moment of the distribution—that is, mean current employment—as well as log aggregate productivity,\(^{51}\)

\[
\log N' = \theta_0 + \theta_N \log N + \theta_p \log p'.
\]  

(63)

Given the forecast of \(N'\) implied by (63), firms can use (62) to forecast future wages, and thereby solve for its optimal employment policy \(f_L(n), X(n), U(n)\).

Aggregation now follows from Proposition 1, which maps the microeconomic policy rules to the law of motion of the distribution. This enables us to simulate a time series of aggregate employment (conditional on the forecast (63)) without the need to simulate hundreds of thousands of individual firms. We then estimate (63) on the simulated time series, update the coefficients \(\{\theta_0, \theta_N, \theta_p\}\), and repeat until convergence.\(^{52}\) We set the elasticity of labor supply to unity, \(\eta = 1\), in line with Chang and Kim (2006) and Kimball and Shapiro (2010). The intercept \(\psi\) is set so that mean equilibrium employment remains near 20.

Figure B1 presents the impulse response of aggregate employment implied by the baseline calibration of the model, and compares it with its frictionless counterpart. Recall that Proposition 2 suggests that approximate aggregate neutrality follows for any configuration of the aggregate state \(\Omega\). It follows that it also will hold for the equilibrium path of \(\Omega\). Consistent with this, the impulse response in Figure B1 is indistinguishable from the frictionless case. The forecast equation (63) is thus very accurate—estimating (63) on model-generated data yields an \(R^2\) in excess of 0.99999—and its estimated coefficients are very close to the frictionless model’s, given the calibration of the labor supply elasticity.\(^{53}\)

\(^{50}\)To derive this, one can imagine a large household comprised of workers with heterogeneous labor supply preferences (Mulligan, 2001). If utility is separable and linear in consumption, labor supply simplifies to (62). Alternatively, one can assume that a large household with identical members chooses an employment rate under Greenwood, Hercowitz, and Huffman (1988) preferences. In either case, the marginal utility of wealth is absent from (62), easing computational burden as there is only one price (the real wage) to track.

\(^{51}\)The forecast of \(\log p'\) follows directly from its exogenous law of motion.

\(^{52}\)Although this quasi-analytical method is preferred, we find virtually the same results when we use a “brute-force” simulation method that solves for and simulates the discretized joint distribution of \(n, x\).

\(^{53}\)Specifically, we estimate \(\hat{\theta}_0 = 2.9821, \hat{\theta}_p = 0.735, \) and \(\hat{\theta}_N = 0.00406\). In the frictionless model, the elasticity with respect to aggregate productivity is the same.
1. Baseline parameterization

2. Lower idiosyncratic dispersion, $\sigma_x = 0.2$

Figure B. Dynamic response in presence of equilibrium wage adjustment

Figure B2 performs the same exercise for the small-$\sigma_x$ parameterization in Figure 4B. Recall that the latter induced a small hump shape in the impulse response in the presence of a constant market wage. We anticipate that the impulse response in market equilibrium will exhibit less of a hump shape, consistent with King and Thomas (2006). Intuitively, firms now recognize that, if labor demand increases in the future (as in the fixed-wage impulse response), the wage will increase then, too. Some firms will therefore bring forward labor demand in order to pay a lower wage. This attenuates the hump shape. Figure B2 confirms this argument. Employment remains elevated in the first period after the shock, but the hump shape has largely vanished. Moreover, the model’s impulse response closely tracks the dynamics of the frictionless model beginning in the second period after shock.

B.1 References


\footnote{The slight hump shape is reflected in the estimated least squares coefficients in (63). These are now $\hat{\theta}_0 = 2.8316$, $\hat{\theta}_p = 0.7008$, and $\hat{\theta}_N = 0.053$. The goodness of fit is, again, excellent: the $R^2$ is 0.999997.}