Job security, asymmetric information, and wage rigidity*

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Abstract

A frictional labor market is considered in which firms commit to wage contracts but cannot commit to retain workers. In a baseline model optimal wage contracts ensure incumbents are not replaced in downturns. These contracts display a degree of wage rigidity for new hires when productivity falls which magnifies the response of unemployment to negative shocks. A key extension is to allow for asymmetric information about productivity. This model generates novel predictions for wage behavior that are supported in our German administrative data; wage rigidity is magnified in downswings and wages respond to forecasts of productivity rather than actual values.

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1 Introduction

The behavior of real wages over the business cycle is critical to understanding the mechanisms that drive employment and output fluctuations. The procyclicality or otherwise of real wages was the subject of considerable debate following the publication of Keynes's (1936) *General Theory*, and it remains a subject of considerable interest.[1] In this paper, we develop a model that has implications for the cyclicality of real wages and for output volatility, but one that emphasizes *asymmetric* wage responses to different phases of the business cycle. Our model exhibits equilibria where partial "equal treatment" is at play. Here the wages of new hires are equal to those of existing (incumbent) workers in recessions even though it would benefit firms *ex post* to pay new hires less. The implication is that if there is a reason for wages of incumbents to be rigid — here, risk aversion — this will be transmitted to the wages of new hires in recessions.

This paper starts out with a baseline model — building on the approach of Menzio and Moen (2010) — with the above characteristics, but then goes on to develop an extension that allows for asymmetric information about the state of nature (productivity). This extended model is a key innovation of our paper; it generates novel implications for wages, implications which find support in our data. The extended model assumes that firms are better informed than workers about the aggregate state so that contracts cannot be conditioned on aggregate variables. It results in wages that may be fully rigid downwards (to be precise: wages may fall but the rate of fall will be independent of the severity of negative shocks), thus further amplifying the variability of unemployment and vacancies. We show that it is the *interplay* between endogenous equal treatment in bad states and asymmetric information that leads to this result; without equal treatment, introducing asymmetric information has no impact on allocations.

A rough intuition for the result is as follows. Equal treatment in downturns arises from the desire to insure incumbents against losing their jobs (see below). But equal treatment implies that the new-hire wage will be above what firms would otherwise wish to pay. If equilibrium wages were to fall with productivity across downturn states, firms have an incentive to exaggerate the severity of downturns; doing so would allow them to lower the (common) wage and — for new hires — bring it closer to what would be optimal absent equal treatment.

[1] See Galí (2013) for a comparison of the cyclicality of real wages in the General Theory and in New Keynesian Models and, e.g., Pissarides (2009) for a discussion of more recent empirical evidence in the context of the "unemployment volatility puzzle" (Shimer, 2005; Costain and Reiter, 2008).

Hence the only incentive-compatible contract may involve a (large) range of shocks in downturns for which wages of incumbents and new hires are not only equal to each other, but do not vary with the severity of the shock.

An outline of the paper is as follows. Following a brief literature review section 3 outlines our baseline symmetric information model. We characterise equilibrium wage contracts assuming that optimal wage contracts always satisfy a condition that new-hire wages are at least as high as those for incumbents. We then analyze when it is optimal for a firm to satisfy this condition. Section 4 contains the key innovation in our paper. There we extend the model to allow for workers being asymmetrically informed about the state. In Section 5, we test certain predictions of the model using German administrative data. As we have noted the predictions of the asymmetric information version of the model are both novel and striking and this data provides some support for these features. A key implication of the asymmetric information model is that in downturns wages should be better related to the forecasted severity of the recession rather than its actual severity and this is what we find in the data. Section 6 contains concluding comments.

2 Relationship to the Literature

The baseline, symmetric information, version of our model builds on and follows the logic of Menzio and Moen (2010). In their paper, overlapping generations of two-period lived firms interact with infinitely lived workers in the context of a frictional labour market, but where employment dynamics are driven by firm entry (each firm employs a fixed number of workers).[2]

As in their model, equal treatment in downturns arises to protect incumbents from the risk of being replaced by cheaper outsiders. The ex ante costs to firms of compensating workers for this risk may more than offset any ex post benefits from violating it. In more detail: Firms can commit to current and future wages but not to employment. In particular, they cannot commit *not* to lay off a worker ("at will"labor contracts). If the wage of new hires is below that of incumbents, the firm will have an incentive to replace its incumbents if it can find suitable applicants, and depending on matching probabilities, there will be a possibility that an incumbent is replaced. Hence firms can effectively commit to

[2]We expand on the main differences in Section 3, but rather than firm entry being the driver of employment fluctuations, we assume a fixed number of firms operating subject to decreasing returns to scale.

not replace incumbents only by setting new-hire wages at least as high as those of incumbents. Workers will have a preference for a contract with job security, and overall employment costs may be lower with such contracts.[3]

In downturns, the desire to smooth wages of incumbents gets transmitted, by equal treatment, to new-hire wages. This leads to a degree of downward rigidity as firms trade-off smoothing with taking advantage of a slacker labour market for new hires. When productivity is high there is no problem in paying a higher wage to new hires than to incumbents, so the rigidity operates only in a downward direction. New-hire wages are allocational in our two-period model, so the downwardly rigid wage affects hiring and increases the variability of both unemployment and vacancies in response to productivity shocks. Equal treatment can also lead to amplified unemployment fluctuations in competitive models (e.g., Thomas, 2005; Snell and Thomas, 2010). See Gertler and Trigari (2009) for a somewhat related mechanism within a search-matching model with staggered Nash bargaining rather than optimal contracting as employed here. In our asymmetric information extension, wages of new hires and incumbents remain linked to an extent also in upswings due to incentive compatibility constraints; nevertheless new-hire wages remain allocational.

Our emphasis is on situations when it is optimal to avoid replacement, and we consider conditions under which this holds. Our empirical results also suggest that firms do not exploit downswings to undercut incumbents. However existing empirical work is ambiguous on the matter. Acharya and Wee (2018), e.g., argue, using US data, that a significant amount of replacement hiring (when incumbents are replaced as opposed to quitting) occurs by comparing total numbers of hires in excess of job gains at firms over time with data on quits. They also argue that evidence in Michaels et al. (2016), who focus on firms with zero net employment changes, may be supportive of this point of view. In their theoretical model, firms may replace an incumbent if a higher productivity match for the position happens. Because they take a Nash bargaining approach to wage determination, replacement will occur as the firm will benefit from some of the extra match surplus, so, unlike here, the firm cannot commit to a wage policy that prevents replacement. By contrast, the findings of Bachmann et al. (2020) for Germany suggest that replacement hiring, as defined by our theory, seems not to be significant in Germany. If it exists, it would imply that worker churn, due

[3] This type of argument was also made in Snell and Thomas (2010) in the context of a perfectly competitive labour market. Menzio and Moen's (2010) model, however, concerns a frictional labour market, and we follow their approach.

to separations and hires into and out of non-employment, increases in recessions. However, Bachmann et al. (2020) show that cyclical variations in worker churn — which is actually procyclical — is accounted for almost wholly by job-to-job transitions rather than by transitions to and from non-employment.

Menzio (2005) considers an asymmetric information bargaining model in which firms are informed about the current state of productivity and workers are not, and it exhibits equal treatment. There are transitory shocks, but if these are not very persistent, the firm does not respond to them; the cost of responding to a positive shock involves paying all workers extra because of equal treatment, while the benefit in terms of additional hiring and retention is smaller when the shock is not expected to persist for long. While we analyze a contracting model, the logic underlying rigid wages under asymmetric information is somewhat related.

Other related work in which asymmetric information amplifies fluctuations includes Kennan (2010), who develops a model of procyclical information rents to firms: if a privately observed (to firms) component of match surplus has more dispersion when the aggregate state of the economy is better, and bargaining leads to an outcome in which firms capture the informational rent, wages are again relatively rigid, and procyclical rents to employer mean that employment fluctuations are magnified. Moen and Rosen (2011) analyze a model of moral hazard (unobservable worker effort) and competitive search and show that it introduces a counter-cyclical element to rents accruing to workers relative to a standard search-and-matching model, enhancing fluctuations in employment over the cycle. However, see also Guerrieri (2007) for a model in which workers have private information about match characteristics but which exhibits little amplification. Bruegemann and Moscarini (2010) derive a bound on extra employment amplification that can arise in frictional labor markets when there is acyclicality in worker *rents* (surplus relative to outside options) rather than wages per se, which is weaker than wage acyclicality when, as usual, outside options are procyclical. They argue that standard asymmetric bargaining models (where there is asymmetric information about match characteristics rather than an aggregate state) may achieve rent acyclicality but will not exceed their bound.

For the empirical results, we attempt to identify asymmetric responses of real wages to business cycle up- and downswings. This is in contrast to the empirical literature on wage stickiness, which typically has looked for evidence of downward real (and also nominal) rigidity by comparing empirical wage-change distributions with notional distributions, i.e., an attempt to capture how wage changes will be distributed in the absence of downward rigidities. An example is Dickens et al. (2007), who summarize results from the *International Wage Flexibility Project*. They use data from 16 OECD countries and find evidence of wage changes clustered around the expected inflation rate and fewer than the expected number of changes below that rate. This and similar evidence points to the existence of some real downward rigidity in individual wage changes in ongoing employment relationships. Our approach differs in that we focus on the real wages of new hires and incumbents separately (the former are omitted by construction in the usual approach) and look at how these wages respond to different phases of the cycle. See Basu and House (2016) for a recent survey of the literature relevant to downward nominal rigidity, which also considers how real labour costs are impacted by rigidities.

Recent evidence from a study of 15 European Union countries by Galuscak et al. (2012) suggests that new-hire wages are intimately related to wage structures that already exist in the firm; moreover, this relationship is stronger in periods of labour market slack, which is a feature of the equilibrium we derive here. Galuscak et al. (2012) argue that fairness and incentive issues are important in leading to this linkage. This is consistent with evidence collected by Bewley (1999), who argue that internal equity considerations make it difficult for firms to employ new hires at a wage lower than that paid to incumbents. Gertler and Trigari (2009) estimate the cyclicality of hiring wages in the U.S. by using Survey of Income and Program Participation data and argue that wages of new hires do appear to be more procyclical than those of ongoing employees. However, using the same data, Gertler et al. (2020) find that it is the composition of match quality that explains the greater wage flexibility for new hires from unemployment.

In Snell et al. (2018), we also examined evidence of downward real rigidity in German data. The model tested in that paper does have worker insurance but no search frictions; the labour market there is competitive. It predicts downward rigidity for both incumbents and new hires in bad states but — contrary to the current paper — also predicts equal treatment in upswings. We return to this earlier empirical work in Section 5 where we compare and reconcile it with the findings in the current paper.

3 The Baseline Model (Symmetric Information)

3.1 Model with No-Undercutting Condition Imposed (Restricted Model)

There are two periods $t = 1, 2$, and a large number of identical firms and workers.[4] Each firm and worker lives for both periods, and the ratio of workers to firms equals S. We identify each firm with the entrepreneur who owns it; entrepreneurs do not supply labour. In each period, each firm operates a decreasing returns technology that produces a perishable good, with production function $f(n; x)$, where n is the current number of workers employed at the firm, which we treat as a continuous variable, $x \in X$ is a productivity shock observable at the start of the period, and derivatives with respect to the first argument are $f' > 0$, $f'' < 0$, with $f(0; x) = 0$. Hours per worker are not variable. We assume that $x = x_0$ is fixed at $t = 1$, but at $t = 2$, x is a random variable, common across firms, with finite support. Henceforth, x without a 0 subscript will refer to the second period productivity shock. Each worker has a per-period utility of consumption function $v(c)$, with $v' > 0$ and $v'' < 0$. Workers cannot borrow or save, so they consume all their current income; we assume for simplicity that there is no discounting of the future by workers. Entrepreneurs, on the other hand, are risk-neutral, but they also have a zero discount rate (nothing depends on this, provided that discounting is symmetric). A worker who is unemployed in any period receives an income of b.

A firm has a wage policy $\sigma = \left(w_1, (w_{2,i})_{i=I,N}\right)$ to which it commits, where $w_{2,I}$ is the second-period wage paid to incumbents, $w_{2,N}$ that paid to new hires in period 2, and $w_{2,i}$ may be random (state contingent). For the moment we assume that it is optimal to satisfy the *no-undercutting condition* $w_{2,N}(x) \geq w_{2,I}(x)$ and treat it as an exogenous constraint. We refer to this as the *restricted model.* We relax this below where we analyse circumstances in the unrestricted model under which it is optimal to satisfy the condition, and those where it is not; to avoid further cluttering the exposition we defer details of this part of the model (modeling the costs of violating the condition) until Section 3.3. Given this condition, a worker who accepts a contract at $t = 1$ suffers only exogenous separation risk from the firm at the end of the first period, with probability δ. In this case, they will be in the same position as a worker who failed to gain employment in the first period; in the second period, such unattached workers

[4] Formally, we will treat these as measures.

seek work.

At the start of each period (in period 2, after x is observed), search and matching occur (see Figure 1). We assume directed search (see Moen, 1997; Acemoglu and Shimer, 1999; Rudanko, 2009). Briefly, an unemployed worker can apply for one job at a single firm in each period.[5] We rule out on-the-job search so that at $t = 2$, a worker cannot apply for a job if he or she is already employed. We identify the 'type' of a job with the utility V a successful applicant obtains from it. The application succeeds with probability $p(\theta(V))$, where $\theta(V)$, "the expected queue length for the job," is the ratio of applicants to jobs of type V, that is, the inverse of labor market tightness.[6] The function $p(\cdot)$ is assumed to be strictly decreasing, differentiable and such that $p(0) = 1$, $p(\infty) = 0$. Correspondingly, the firm fills a job of type V with probability $q(\theta(V))$ where $q(\cdot)$ is strictly increasing, and satisfies $q(\theta) = p(\theta)\theta$, $q(0) = 0$ and $q(\infty) = 1$. Moreover, denoting the elasticity of q with respect to θ by $\epsilon_q(\theta)$, $q(\theta)\epsilon_q(\theta)/(1-\epsilon_q(\theta))$ is assumed to be a decreasing function of θ.[7]

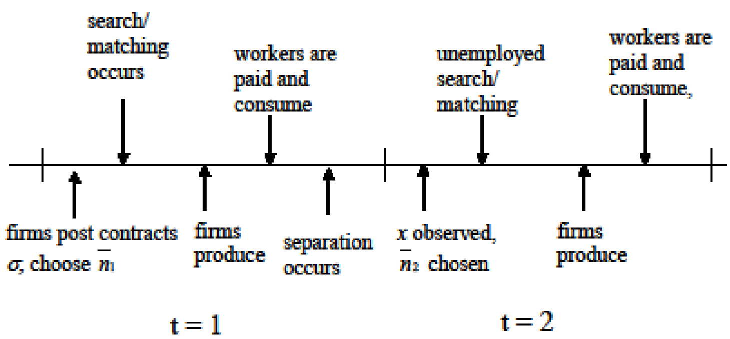

Figure 1: Timeline

Simultaneously with committing to a wage policy at the start of $t = 1$, firms choose how many new jobs $\overline{n}_i$ to create in period $i = 1, 2$, at a cost of $k > 0$ per job; $\overline{n}_2$ depends on the shock x. Unfilled jobs from the first period 'die' at the end of the period, along with filled jobs in which exogenous separation

[5]We do not consider search intensity on the worker side to be a choice variable. See, e.g., Choi and Fernández-Blanco (2018), who consider optimal policy in a two-period directed search model with contract posting, as here, where search intensity depends on unemployment risk amongst other things.

[6]For the moment, we suppress other arguments of $\theta(\cdot)$ corresponding to the economic environment. The determination of $\theta(V)$ is discussed below.

[7]Menzio and Moen (2010), who also assume this, point out that many standard matching processes satisfy these assumptions.

occurred. The implication is that employment at the firm in period i will increase by $q(\theta(V))\overline{n}_i$.[8]

Let Z_1 be the lifetime utility of a worker at the search stage in period 1 and $Z_2(x)$ be that of a worker in period 2 searching for work in state x. Z_1 and Z_2 are the endogenous variables determining the economic environment the firm faces. Define $Z = \left(Z_1, (Z_2(x))_{x\in X}\right)$. The value to a worker at $t = 1$ from being employed by a firm with wage policy σ is then

$$V_1(\sigma; Z) := v(w_1) + E[\delta Z_2(x) + (1-\delta)v(w_{2,I}(x))], \tag{1}$$

where E denotes the expectation.[9]

Let U_1 be the lifetime utility of a worker at $t = 1$ who fails to get a job:

$$U_1(Z) = v(b) + E[Z_2(x)],$$

as currently, the worker receives b and is able to search next period. Given U_1 and Z_1, the expected queue length for a job offering V_1 is assumed to satisfy:

$$\theta_1(V_1, Z_1, U_1) = \begin{cases} \theta : p(\theta)V_1 + (1-p(\theta))U_1 = Z_1, & \text{if } V_1 > Z_1 \\ 0, & \text{if } V_1 \leq Z_1 \end{cases}. \tag{2}$$

The idea is that if the value of the job to a successful applicant, V_1, is greater than the value of search, Z_1, the expected queue length is driven up to the point where workers are indifferent between applying for the job and searching somewhere else, and vice versa. The expected queue length for the job will be zero if the value of the job is less than (or equal to) the value of search.

[8]Our base model differs from Menzio and Moen (2010) in the following principal respects. First, our workers are two-period lived rather than infinitely lived (firms in Menzio and Moen (2010) are two-period lived), and we have a two-period horizon — an extension to multiple periods is straightforward and discussed below in Footnote 19. Second, rather than having firms of a fixed size (number of jobs) with constant productivity per filled job and free entry of firms, we suppose that there are a fixed number of firms, each with a decreasing returns to scale technology. The supply of jobs then varies not with variations in the number of firms entering the market but with firms' choices about how many jobs to create in each period. The fixed cost per job created replaces Menzio and Moen's (2010) assumption of a fixed cost incurred per firm that enters. Overall our model admits more tractability; in particular we are able to evaluate the model's response to standard productivity shocks whilst Menzio and Moen's (2010) set up only readily admits analysis of responses to MIT shocks.

[9]To avoid complicating the exposition, we will ignore the possibility that at the optimal period-2 wage, the firm will prefer not to hire at all, and to dismiss some of its incumbents. This situation will arise if $w_{2,I} > f'((1-\delta)n_1; x)$. In our simulations, parameters are chosen so that this scenario does not arise: We will assume throughout that positive hiring occurs in equilibrium. Given average annual turnover rates of around 30% in the U.S., e.g., this assumption is not restrictive for any reasonable parameterization.

For a worker seeking work at $t = 2$, the value from being employed at the wage $w_{2,N}$ is $v(w_{2,N})$, so the expected queue length for period-2 firms and workers for a job with wage $w_{2,N}$ is

$$\theta_2(w_{2,N}, Z_2) = \begin{cases} \theta : p(\theta)v\left(w_{2,N}\right) + (1 - p(\theta))v\left(b\right) = Z_2, & \text{if } v\left(w_{2,N}\right) > Z_2 \\ 0, & \text{if } v\left(w_{2,N}\right) \leq Z_2 \end{cases}. \tag{3}$$

A firm's profit is

$$F\left(\sigma; \overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}; Z\right) = \left(f\left(n_1; x_0\right) - w_1 n_1 - k\overline{n}_1\right) + \tag{4}$$
$$E\left[\left(f\left((1-\delta)n_1 + n_2; x\right) - w_{2,I}(1-\delta)n_1 - w_{2,N}n_2 - k\overline{n}_2\right)\right]$$

where n_i is the number of new hires in period i and is given by $n_i = q\left(\theta_i\right)\overline{n}_i$, $i = 1, 2$, where θ_i depends on σ, as given by $\theta_1(V_1\left(\sigma, Z\right), Z_1, U_1\left(Z\right))$ in (2) and $\theta_2(w_{2,N}, Z_2\left(x\right))$ in (3) above.

Competitive Search Equilibrium

We define an equilibrium for the restricted model as follows:

Definition 1 *A symmetric equilibrium in the restricted model with positive hiring consists of search values* $Z = \left(Z_1, \left(Z_2\left(x\right)\right)_{x\in X}\right)$, *and a wage policy* σ *satisfying* $w_{2,N}(x) \geq w_{2,I}\left(x\right)$, $x \in X$, *and job creation plan* $\left(\overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}\right)$ *with the following properties:*

(i) *Profit maximization*: For all $(\sigma'; \overline{n}_1', (\overline{n}_2'\left(x\right))_{x\in X})$ satisfying $w_{2,N}'(x) \geq w_{2,I}'\left(x\right)$, $x \in X$,

$$F((\sigma; \overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}); Z) \geq F\left(\sigma'; \overline{n}_1', \left(\overline{n}_2'\left(x\right)\right)_{x\in X}; Z\right);$$

and

(ii) *Consistency*: $\theta_1\left(V_1\left(\sigma, Z\right), Z_1, U_1\right) = S/\overline{n}_1$, and, for all x, $\theta_2(w_{2,N}, Z_2\left(x\right)) = S_2/\overline{n}_2\left(x\right)$ where $S_2 := \left(\left(1 - p\left(S/\overline{n}_1\right)\right) + \delta p\left(S/\overline{n}_1\right)\right)S$ is the number of workers (per firm) seeking work in period 2.

3.2 Characterization of Equilibrium Contracts in the Restricted Model

We proceed heuristically.[10] This paper adopts the assumption throughout that firms cannot commit to employment (retention) of incumbents who are not exogenously churned. In the restricted model retention is achieved because of the no-undercutting condition. It is nevertheless useful to compare the current model's equilibrium wage dynamics with the standard case where firms *can commit* to retain incumbents who are not exogenously churned, even when $w_{2,N} < w_{2,I}$. We refer to the equilibrium from this alternative model as the *commitment to retain or CTR equilibrium*. In that model, firms would completely insure risk-averse period-1 hires so $w_{2,I} = w_1$ in each state, and $w_{2,N}$ would be set (in combination with job creations) so as to minimize the total cost of hiring each worker.

We show that in any period-2 state where the new-hire wage $w_{2,N}$ is below w_1, $w_{2,N}$ will be above the corresponding CTR level: because the no-undercutting condition binds so $w_{2,I} = w_{2,N}$, firms have to trade-off offering less insurance to period-1 hires with cutting $w_{2,N}$ as far as they would like to. Hence the inability to commit to retain incumbents leads to a reduction in downwards wage flexibility. By contrast when $w_{2,N}$ is above the period 1 wage then it will be at the corresponding CTR level as there is no trade-off to dampen wage increases.

In period 2 in any state x, given n_1 and w_1, it can be shown that the firm must locally maximize profits plus weighted incumbent utility. In particular, it must maximize

$$f\left((1-\delta)n_1 + n_2; x\right) - w_{2,I}(1-\delta)n_1 - w_{2,N}n_2 - k\overline{n}_2 + \\ \left(1/v'\left(w_1\right)\right) n_1 \left((1-\delta)\, v\left(w_{2,I}\right) + \delta Z_2\left(x\right)\right), \tag{5}$$

with respect to $\overline{n}_2$, $w_{2,N}, w_{2,I}$, $w_{2,N} \geq w_{2,I}$, where $n_2 = q\left(\theta_2\left(w_{2,N}, Z_2\left(x\right)\right)\right)\overline{n}_2$ $=: \tilde{q}\left(w_{2,N}, x\right)\overline{n}_2$. We write $\tilde{q}' \equiv \partial\tilde{q}/\partial w_{2,N}$. Note that the last term in (5) includes the continuation utility of an incumbent, taking into account the separation possibility and multiplied by the number of incumbents. The intuition here is that any change that affects the utility of the firm's old workers can be offset by a change in the first period wage, leaving V_1 unchanged (and, hence, n_1). Multiplying the utility change by the inverse of first-period marginal utility then converts it (for a small change) to the first-period wage savings per worker.

[10] Formal proofs are provided in Online Appendix B.

There are two cases to consider:

(A) If the *no-undercutting condition*, $w_{2,N} \geq w_{2,I}$, is not binding, then differentiating (5) with respect to $w_{2,I}$,

$$(1-\delta)n_1 = n_1 \left(1/v'\left(w_1\right)\right)\left((1-\delta)\, v'\left(w_{2,I}\right)\right), \tag{6}$$

so that $w_1 = w_{2,I}$. Intuitively, the firm should stabilize the wages of the first period hires if there is no cost of doing this. In this case, also differentiating with respect to $w_{2,N}$, we obtain

$$f'\left((1-\delta)n_1 + n_2; x\right) q'\overline{n}_2 - w_{2,N} q'\overline{n}_2 - q\overline{n}_2 = 0, \tag{7}$$

and simplifying:

$$f'\left(n\right)\tilde{q}' - w_{2,N}\tilde{q}' - q = 0,$$

where we write $n \equiv (1-\delta)n_1 + n_2$ for total period-2 employment. Finally, differentiating with respect to $\overline{n}_2$,

$$f'\left(n\right) = w_{2,N} + k/q. \tag{8}$$

We can combine these latter two to obtain

$$q^2 \left(\tilde{q}'\right)^{-1} = k. \tag{9}$$

Intuitively, in order to increase employment by one unit, the firm could open $1/q$ jobs at a cost of k/q. Alternatively a wage increase of $1/\left(\overline{n}_2\tilde{q}'\right)$, holding the number of jobs constant, accomplishes the same result by increasing the queue length and, hence, the probability that each existing job is filled, at a cost of $q\overline{n}_2 \times 1/\left(\overline{n}_2\tilde{q}'\right) = q/\tilde{q}'$. The two must be equal in equilibrium so that (9) follows.

The wage $w_{2,N}$ that corresponds to the cheapest way of hiring n_2 workers (taking into account the number of jobs that must be created) traces out the *restricted model quasi-supply* curve of labor. This is the locus of values for n_2 and $w_{2,N}$ compatible with (9). In the proof of Proposition 1 it is shown that it is a positively sloped locus: when equilibrium n_2 is higher, it is more difficult to fill each job because the labor market is tighter (θ_2 is lower, so $k/q\left(\theta_2\right)$ is higher). This makes wage increases, as a way to fill jobs, more attractive than creating extra jobs, and $w_{2,N}$ rises until the two methods cost the same. The locus is independent of the revenue generated from a filled job. In this region it also corresponds to the solution to the first-order conditions in the CTR model (where firms can commit to employment). We refer to the latter as the *CTR quasi-supply curve*. We can combine this with the downward sloping (8), which is a standard labor demand equation, where the unit cost of increasing employment $k/q\,(\theta_2)$ (itself increasing as n_2 increases)[11] is added to the wage. The intersection yields a unique equilibrium for each value of x.[12] As x varies, only the labor demand curve shifts. Denote the solution of (8) and (9) by $\left(w_{2,N}^{CTR}\,(x, w_1, n_1)\,, n_2^{CTR}\,(x, w_1, n_1)\right)$, where the CTR-superscript indicates that this is also the solution to the first-order conditions in the CTR model.[13]

Since in this case, $w_{2,N} \geq w_{2,I} = w_1$, we conclude that the intersection of (8) and (9) occurs at or above w_1.

(B) If, on the other hand, $w_{2,N} \geq w_{2,I}$ is binding at the optimum (when productivity is sufficiently low), the intersection of (8) and (9) occurs at a wage below w_1, but the wage can be shown to be above $w_{2,N}^{CTR}\,(x, w_1, n_1)$, while employment is below $n_2^{CTR}\,(x, w_1, n_1)$. In the proof, it is shown that $k < q^2/\tilde{q}'$. The unit cost of increasing employment through creating extra jobs, k/q, is lower than that through increasing wages, $q/\tilde{q}'$, so it would be cheaper to cut wages and increase jobs; however, this is not done because the wage cut has a negative externality on incumbents' wage smoothing. More intuitively, if productivity is low enough that the equilibrium hiring wage in the absence of the condition $w_{2,N}^{CTR}$ is below w_1, then the no-undercutting condition will be violated (recall that $w_{2,I}^{CTR} = w_1$). To satisfy the condition, $w_{2,I}$ must be cut, which is costly because it reduces wage smoothing, so firms are less willing to let wages fall. Thus, below w_1, the equilibrium lies above the CTR quasi-labor-supply curve.

Consequently, taking w_1 as given, the restricted model quasi-supply curve coincides with the CTR one above w_1, but below w_1, the curve lies above the CTR curve. Equilibrium again occurs at the intersection with the labor demand curve. In Figure 2, a situation where the crossing point occurs below w_1 is illustrated.[14] The equilibrium values are at point A, rather than at the CTR solution. If x is sufficiently high such that the intersection occurs above w_1, then

[11]As n_2 increases, we must have $p\,(\theta)$ increasing from $n_2 = p\,(\theta)\,S_2$, and hence, θ has fallen as $p' < 0$; thus $q\,(\theta)$ falls, given that $q' > 0$.

[12]The positions of these two curves depend only on x and n_1, which implies the value of S_2.

[13]This will not correspond to the full equilibrium of the CTR model as n_1 and w_1 will in general be chose differently, hence the comparison is with a CTR model with these same choices in period 1 (and the same x in period 2). If the states in which $w_{2,N} \geq w_{2,I}$ binds have low probability, n_1 and w_1 will be approximately equal in both models.

[14]In simulations of the constrained quasi-supply curve, as n_2 falls, we find that wages eventually start to increase. The intuition is that the number of new hires falls sufficiently low such that the desire to insure incumbents dominates and the wage approaches w_1 as n_2 goes to zero.

the equilibrium will be at the CTR solution, $\left(w_{2,i}^{CTR}\left(x, w_1, n_1\right), n_2^{CTR}\left(x, w_1, n_1\right)\right)$. The proposition summarizes the discussion.

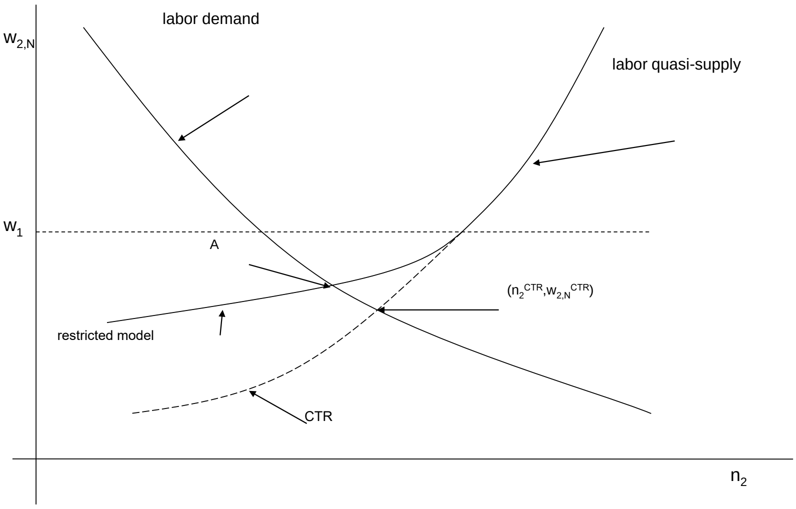

Figure 2: Restricted model quasi-supply

Proposition 1 *(a) If equilibrium hiring wages in any state in period* 2 *are below period*-1 *wages,* $w_{2,N} < w_1$*, we have* $w_{2,N} > w_{2,N}^{CTR}\left(x; w_1, n_1\right)$ *and* $n_2 < n_2^{CTR}\left(x; w_1, n_1\right)$*: the wage for new hires is higher and employment is lower than they would be in the CTR model;* [15] *moreover,* $w_{2,I} = w_{2,N} < w_1$*. Otherwise, (b) wages and employment are at the CTR levels:* $w_{2,N} = w_{2,N}^{CTR}\left(x; w_1, n_1\right)$ *and* $n_2 = n_2^{CTR}\left(x; w_1, n_1\right)$*, with* $w_{2,I} = w_1$*. Case (a) occurs when the labor demand curve intersects the CTR quasi-supply curve below* w_1*; otherwise, case (b) occurs.*[16]

Wages are allocational[17] in period 2 so that the flatter quasi-supply in the region where there is downward pressure on wages will also imply more variable

[15] If firms were not constrained by the no-undercutting condition in such a state, unless the state had a negligible probability, then the equilibrium two-period contract may be different, that is, w_1 and n_1 may differ. The proposition concerns the implied values of $w_{2,N}^{CTR}$ and n_2^{CTR} in a hypothetical equilibrium that has the same period-1 values.

[16] Formal proof is provided in Online Appendix B.1.

[17] I.e., firms hire until the marginal product net of the hiring cost (k/q) is equal to the new-hire wage.

employment.[18,19] The result is unchanged if there is symmetric discounting. If discounting is asymmetric, then the reference wage in period 2, which determines the regime (and $w_{2,I}$ when no undercutting does not bind), differs from w_1, but otherwise the proposition extends.

3.3 Unrestricted model: When is it optimal to satisfy the no-undercutting condition $w_{2,N} \geq w_{2,I}$?

Here we drop the working assumption that optimal wage contracts are such that no undercutting occurs, and flesh out the implications for wages when undercutting can occur. As in our baseline model we continue to assume that the firm cannot commit *not* to replace workers by cheaper new hires.[20] We then analyze circumstances under which a firm will want to satisfy the condition $w_{2,N} \geq w_{2,I}$ in order to avoid uncertainty created by employment risk; in short we outline the circumstances in which no undercutting is a feature of the optimal wage contract.

We suppose that employment is "at will", so during the matching stage of the second period (after observing x), the firm can dismiss a worker without compensation; that is, the firm can dismiss a worker after matching with an

[18]E.g., take the matching function $m(u,\nu) = uv/\left(u^l + \nu^l\right)^{1/l}$, where u is the number of workers searching and ν is the number of vacancies, where we set $l = 0.5$ (Hagedorn and Manovskii (2008) calibrate $l = 0.407$), with a log production function subject to uniformly distributed multiplicative productivity shocks, constant relative risk-aversion utility and a coefficient of risk aversion of 2, $\delta = 0.1$ (approximate annual separation rate in our German data), $\beta = 0.9$, a replacement rate of 43%, and we calibrate k to yield an average period-2 unemployment rate of 7.5% with a distribution of shocks with coefficient of variation of 0.25. The standard deviation of unemployment in the region where the no-undercutting condition is binding is approximately twice that in the CTR model. The effect is smaller, however, under alternative parameterizations. With a Cobb-Douglas matching technology with $p(\theta) = M\theta^{\eta-1}$, $q(\theta) = M\theta^{\eta}$, where $M = 1/10$ and $\eta = 1/2$ (this is the same specification used in Menzio and Moen's (2010) example) and $v(c) = c^{0.5}$, and setting $\delta = 0.3$ (appropriate for US annual data), we obtain a much smaller increase of approximately 25%. This result is partly attributable to a lower risk-sharing motive, but the higher separation rate means that in bad states, the incentive to bring in new hires at a lower wage is stronger.

[19]If there are multiple periods (with long-lived firms and workers), Proposition 1 readily extends, where now undercutting is defined in terms of discounted wage costs rather just than the current wage. If no-undercutting in this sense is imposed, incumbents' wages are always no higher than new-hire wages and fall only to maintain this relationship, otherwise remaining constant. Moreover, in downturns, wages do not fall as far as firms would like them to in the following sense: if new-hire wages fall between periods t and $t+1$, they are above the relevant CTR quasi-supply curve at $t+1$; when new-hire wages rise between the two periods, however, they *will* lie on the relevant CTR quasi-supply curve at $t+1$. The principal qualitative difference is that there may be multiple incumbent wages at each date and that the new-hire wage is no longer fully allocative. Details available on request.

[20]Of course in the restricted model replacement does not occur. But this is because of the restriction that incumbent wages in period 2 do not exceed those of new hires.

applicant who can replace the original worker, and the dismissed worker will be unemployed.[21] Specifically, at $t = 2$, suppose that unemployed workers can apply for jobs that are already filled; if there is a successful applicant, the firm can, by at-will contracting, choose whether to replace the incumbent or not. If $w_{2,N} \geq w_{2,I}$ firms will have no incentive to do this (and unemployed workers no incentive to apply for such positions), but for $w_{2,N} < w_{2,I}$ the incentive exists to replace. In the latter case, then, to the extent that the matching process succeeds in selecting a successful applicant for this position, the incumbent is at risk of losing her position. We are assuming there is no cost associated with receiving applicants for filled jobs and that the new-hire wage $w_{2,N}$ applies to any new hire. An incumbent's position is on a par with all other created positions; a filled job is as attractive as an unfilled one from the point of view of an applicant when $w_{2,N} < w_{2,I}$ and is equally likely to be filled by a new entrant.[22]

The expression for profits $F\left(\sigma; \overline{n}_{1,} (\overline{n}_2 (x))_{x \in X}; Z\right)$ is generalized as follows. In the expression for the value to a worker at $t = 1$ from being employed by a firm with wage policy σ, if replacement occurs in some states, that is, if $w_{2,N} < w_{2,I}$, then in such states, the term inside the square brackets in (1) must be replaced by

$$\delta Z_2(x) + (1-\delta) q(\theta_2) v(b) + (1-\delta)(1 - q(\theta_2)) v(w_{2,I}(x)).$$

This expression reflects the additional risk $q(\theta_2)$ to a surviving worker of being replaced by a successful applicant.

Likewise, in any state where replacement occurs, the expression for second-period profit in (4) is replaced by

$$f((1-\delta)n_1 + n_2; x) - w_{2,I}(1 - q(\theta_2))(1-\delta)n_1 - w_{2,N}(n_2 + q(\theta_2)(1-\delta)n_1) - k\overline{n}_2,$$

where $q(\theta_2)(1-\delta)n_1$ is the number of incumbents who are replaced by new hires, and $n_2 = q(\theta_2)\overline{n}_2$ is the number of new hires *into newly created jobs*.

[21] Less relevant is the decision of the worker to quit if we assume a worker can quit without penalty, but will remain unemployed in the second period. This situation implies that the only participation constraint that matters for period-1 hires is the period-1 constraint. An alternative assumption that leads to this implication is that a worker who changes jobs incurs a high mobility cost. In either case we will ignore the worker quit decision.

[22] To be clear, and following Menzio and Moen (2010), in this case, a filled job will attract the same number of applicants as any newly created unfilled job and will have the same probability of a successful applicant being found and, hence, of the incumbent losing his/her position.

3.3.1 No-replacement equilibria

We define a *no-replacement equilibrium with positive hiring* to be an equilibrium of the unrestricted model in which replacement does not occur in any state, or equivalently in which $w_{2,N} \geq w_{2,I}$, and $\overline{n}_2 > 0$, in each state. The definition is as in Definition 1, but without the condition $w'_{2,N}(x) \geq w'_{2,I}(x)$, $x \in X$ in condition (i). That is, if Z, σ, $\overline{n}_{1,}(\overline{n}_2(x))$ is a symmetric equilibrium in the restricted model — where the condition is *imposed* — it remains an equilibrium provided $F(\sigma; \overline{n}_{1,}(\overline{n}_2(x))_{x \in X}; Z) \geq F(\sigma'; \overline{n}'_1, (\overline{n}'_2(x))_{x \in X}; Z)$ where σ' now includes all replacement deviations, wage policies with $w'_{2,N}(x) < w'_{2,I}(x)$ for some x.

We ran simulations based on the parameterization in Footnote 18 to see where replacement deviations do not improve profits, i.e., where the no-replacement equilibrium exists.

As the coefficient of variation of shocks increases — in particular the severity of the bad shock worsens — undercutting becomes relatively more attractive. The optimal new-hire undercut wage falls substantially (and hiring rises) as Z_2 falls, whereas in the putative no-replacement equilibrium $w_{2,N}$ falls much less. This allows the undercutting firm to exploit the state of the labor market in the bad state to a greater extent.

Increasing b, and hence the replacement rate, makes it more likely (i.e., for a wider range of other parameter values) it is optimal to satisfy the no-undercutting condition. This is somewhat counterintuitive in that the downside of undercutting is the risk of replacement — income falling to b — so a lower risk might make undercutting less costly in terms of the period-1 risk premium. However an offsetting factor is how much the new-hire wage can be cut. With b higher the increase in Z_2 makes the optimal new-hire wage in the undercutting deviation higher, more than offsetting any benefit from a reduced risk premium.

Similarly, increasing δ, the rate of turnover, also makes it more likely it is optimal to satisfy the no-undercutting condition. This is again counterintuitive in that the relative importance of new hires in period 2 increases, and so in a bad state the benefit from lower new-hire wages, i.e., undercutting, should increase. However this is offset by the fact that a smaller survival probability reduces the value to stabilizing wages so $w_{2,N}$ will fall in the absence of undercutting. Moreover in equilibrium the additional replacement hiring pushes Z_2 up as θ falls and the labor market tightens. The optimal undercutting wage then is higher when δ is higher, and so there is less to be gained from cheaper new hires.

Reducing job creation costs, k, decreases the likelihood that it is optimal satisfy the no-undercutting condition. In fact the replacement deviation will dominate for k small enough. Intuitively, as the job opening cost falls, θ and $q(\theta)$ fall as more jobs are created. Then, the firm is better off setting $w_{2,N} < w_{2,I}$ and offering full insurance to an incumbent if he/she remains in the firm, but with a small risk of replacement $q(\theta)$. The benefit from a lower new-hire wage is greater than the (very small) risk premium that has to be offered to period-1 hires.

However, consider the limiting case of a competitive labour market, as in Snell and Thomas (2010). In this case, if $w_{2,N} < w_{2,I}$ in some state, all incumbents will be replaced, provided that $w_{2,N}$ is not below the supply price of unemployed workers, as the firm can then hire as many new hires as it wants. Since the supply price of an unemployed worker in period 2 will be at least as great as what a replaced worker would obtain from unemployment, changing the contract so that $w_{2,I} = w_{2,N}$ clearly does not leave the firm worse off, as it faces the same costs at period 2. Period-1 hires will weakly prefer this contract because they are not replaced. Thus, satisfying the no-undercutting condition is weakly dominant (and strictly so if the supply price of the unemployed exceeds what a replaced worker obtains). The reason for this apparent discontinuity at the limit is that although as $k \to 0$ the market in the frictional case becomes competitive (both $p \to 1$ and $k/q \to 0$ in an undercutting equilibrium), and the firm can approximately hire at a going wage,[23] the firm can only find a replacement for an incumbent with a probability tending to zero (rather than a probability of one in the competitive case); in this case, the no-undercutting condition is (optimally) violated.

The parameter space where a no-replacement equilibrium exists is illustrated in Figure 3. We find that there are usually three local maxima to profits for a firm in a putative no-replacement equilibrium. These are the putative equilibrium plan where the condition $w_{2,N} \geq w_{2,I}$ is imposed, an undercutting plan as described above where the firm benefits from bringing in new hires at a wage below that of incumbents, and an undercutting plan where the wage in the bad state falls to match Z_2 which means the firm cannot hire. The latter is an alternative way of committing to retention ($q = 0$); it however can never dominate profits when the condition is imposed, which allows hiring, albeit at a higher wage. The fine dotted line shows the border between areas where either undercutting strategy yields higher profits.[24]

[23] The wage elasticity of employment $\partial(\tilde{q}(w_{2,N}, x)\,\overline{n}_2)w_{2,N}/\partial w_{2,N}\tilde{q}(w_{2,N}, x)\,\overline{n}_2 \to \infty$.

[24] One strategy we have not considered is for the firm to dismiss all incumbents and replace

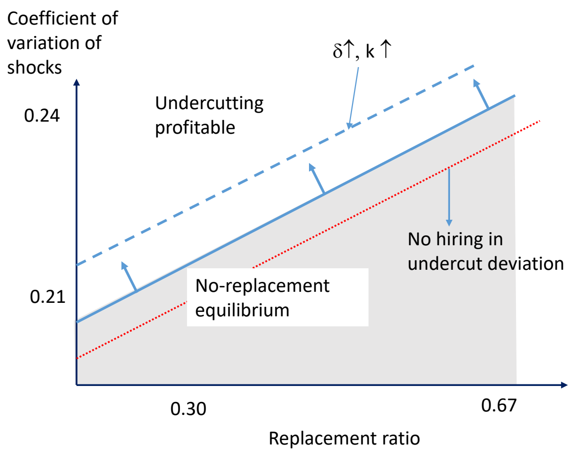

Figure 3: No-replacement equilibrium

3.3.2 Equilibria with replacement

Next, we characterize outcomes when replacement *does* occur in some states in equilibrium.[25] Here we find that, in contrast to the case where replacement does not occur, in downturns new-hire wages are *more* rather then less flexible than the wage from the CTR model, and moreover, the incumbent wage will be completely rigid downwards.[26] The consistency condition for equilibrium must be generalized, so that in any state for which replacement occurs,

$$\theta_2(w_{2,N}(x), Z_2(x)) = S_2/\left(\overline{n}_2(x) + (1-\delta)\, q\left(S/\overline{n}_1\right)\overline{n}_1\right).$$

Proposition 2 *Suppose that replacement occurs in state x in equilibrium. Then, for a given w_1 and n_1, the wage for new hires is lower (and employment is higher) than they would be in the CTR model,*[27] $w_{2,N} < w_{2,N}^{CTR}(x; w_1, n_1) < w_1$*; moreover,* $w_{2,I} = w_1$.[28]

Intuitively, cutting the new-hire wage makes a job less attractive, and there-

them by new hires. This is however dominated by a contract where $w_{2,I}$ is set equal to $w_{2,N}$ and all incumbents are retained, which reduces period 2 costs for the same employment (it saves on hiring costs) and period 1 hires would be better off as $v(w_{2,N}) \geq Z_2$.

[25] The proof of Proposition 1 assumed that there is no replacement in period 2 in *any* state; even with replacement in some states, the statement still holds for non-replacement states x: if there is replacement in some state $x' \neq x$, it modifies the expectation term in (B.1) and (B.4), but they cancel).

[26] We are able to test for these implications against alternatives in Section 5.

[27] See note in Footnote 15.

[28] Formal proof is provided in Online Appendix B.2.

fore the risk of replacement decreases; this positive externality on incumbents makes a wage that is lower than the CTR wage $w_{2,N}^{CTR}$ optimal. The firm should stabilize the wages of the first-period hires because there is no cost of doing this, given that the replacement probability is independent of $w_{2,I}$ whenever $w_{2,N} < w_{2,I}$.

4 Asymmetric Information

In this section we introduce asymmetric information over the period-2 state x into the restricted model, treating the no-undercutting condition as an exogenous constraint as in Section 3.1.[29] We argue that for a wide range of adverse shocks, this may lead to a period-2 wage that is completely rigid for incumbents and, more importantly, for new hires. Moreover, under the assumptions of Proposition 3 below, period-2 wages remain allocational, which leads to enhanced employment variability.

We will assume that in period 2, ongoing hires in a firm can observe only wages $w_{2,N}$ and $w_{2,I}$ but cannot observe x (nor Z_2 so they cannot infer x). Additionally, they cannot observe the total employment or job openings at the firm (we relax this in an online appendix). Equivalently, we assume that such variables are not contractible. The resultant incentive compatibility constraints on the contract imply that the equilibrium contract exhibits a much higher degree of wage rigidity and employment and job opening fluctuations than induced by equal treatment alone.

4.1 Incentive Constraints

As before, assuming that a firm's profit is

$$F\left(\sigma; \overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x \in X}; Z\right) = \left(f\left(n_1\right) - w_1 n_1 - k\overline{n}_1\right) + E[F^{(x)}]$$

where $F^{(x)}$ is period-2 profits in state x and is given by

[29] The analysis of Section 3.3.1 also applies in this case when the variance of shocks is not too large, as an undercutting deviation from a no-replacement equilibrium in the unrestricted model will have approximately the same benefit or cost. Details available on request.

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right) :=$$
$$\left(f\left((1-\delta)n_1+n_2\left(x\right);x\right)-w_{2,I}\left(x\right)(1-\delta)n_1-w_{2,N}\left(x\right)n_2\left(x\right)-k\overline{n}_2\left(x\right)\right)$$

(recall n_i is the number of *new hires* in period i, and is given by $n_i = q\left(\theta_i\right)\overline{n}_i$, $i = 1$, 2, where θ_i depends on σ, as given by $\theta_1(V_1\left(\sigma, Z\right), Z_1, U_1\left(Z\right))$ in (2) and $\theta_2(w_{2,N}, Z_2\left(x\right))$ in (3) above). We now have the firm's maximization problem as $(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X})$ maximizes $F((\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X});Z)$ subject to the incentive compatibility constraints for all x,

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right) =$$
$$\max_{x',\overline{n}'_2}\left\{\left(f\left((1-\delta)n_1+n'_2;x\right)-w_{2,I}(x')(1-\delta)n_1-w_{2,N}(x')n'_2-k\overline{n}'_2\right)\right\}$$

where $n'_2 = q\left(\theta_2\right)\overline{n}'_2$ and $\theta_2 = \theta_2(w_{2,N}(x'), Z_2\left(x\right))$, and the no-undercutting condition is $w_{2,N} \geq w_{2,I}$ for all x. That is, the firm has a menu of wage profiles $\left(w_{2,N}\left(x\right), w_{2,I}\left(x\right)\right)$ to choose from and will optimize job openings, given its choice;[30] incentive compatibility requires that the firm prefers the wage profile associated with the current state to any other.

We now assume that $X \subset R_+$, and that f is differentiable and increasing in x. We can establish the following:

Proposition 3 ***(Asymmetric information)****(i) In the CTR model introducing asymmetric information does not affect the equilibrium. (ii,* wage floor) *Suppose in the restricted asymmetric information model with a single period-*2 *productivity state $\hat{x}$, that there is an equilibrium with no undercutting and the no-undercutting condition binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states, $\hat{x}-\varepsilon$ and $\hat{x}+\varepsilon$ (i.e., with expected value $\hat{x}$), and assuming that there exists $\overline{\varepsilon}$ such that for $\varepsilon \in [0,\overline{\varepsilon})$, the equilibrium is unique and continuous in ε, then period-*2 *wages are* constant *across these states, provided that the perturbation*

[30]These are ex post (after the period-2 state is observed) constraints; for simplicity, we assume that n_1 is contractible. Otherwise, the incentive compatibility constraints should be expressed in terms of an ex ante constraint that requires that should the firm deviate at date 1 (i.e., possibly changing n_1) and in any period-2 state, it cannot increase its discounted expected profit. Since in the latter case, the ex post constraints will also hold, the results will be very similar.

ε is sufficiently small.[31] *Period-2 wages are allocational. (iii,* upward flexibility) *In the restricted asymmetric information model, at the highest $w_{2,I}$, i.e., for $x \in \arg\max_{x'} w_{2,I}(x')$, $w_{2,N}(x) = w_{2,N}^{CTR}(x, w_1, n_1)$ if the no-undercutting condition is not binding, and $w_{2,N}(x) \geq w_{2,N}^{CTR}(x, w_1, n_1)$ otherwise. (iv) $w_{2,I}(x) \leq w_1$, all x.*[32]

Part (i) of Proposition 3 considers the nature of the contract with asymmetric information but in the CTR model. The firm will offer a non-contingent period-2 contract wage to period-1 hires (equal to w_1) and hence there is no benefit from deviating from the optimal hiring wage to period-2 workers.

Part (ii) considers what happens in the restricted model, where asymmetric information now matters: if there are two states close to each other and the no-undercutting condition is binding, then wages are non-contingent; this has direct implications for hires.[33]

While the formal proposition requires the variance of the shocks to be small, simulations suggest that the optimal contract has a fixed period-2 wage for a very wide range of shocks, and where there are multiple shocks. To see the intuition for the proposition, consider the restricted model solution under symmetric information: suppose there are two states x_1 and x_2 at $t = 2$ and that we are in the region where the no-undercutting condition is binding in both states, $w_{2,N}(x) = w_{2,I}(x)$, $x = x_1, x_2$. If the wage varies with the state, say if $w_{2,N}(x_1) = w_{2,I}(x_1) < w_{2,N}(x_2) = w_{2,I}(x_2)$, in state x_2, the firm will prefer to "announce" state x_1: it benefits from paying a lower wage to its existing employees. In addition, because the no-undercutting condition is binding, the optimal wage for new hires (i.e., ignoring the no-undercutting condition) would be lower than at the restricted model solution, and the firm will benefit from a lower wage considering new hires. Therefore, for both reasons, period-2 profits increase. Consequently, the CTR solution will violate incentive compatibility, but a similar logic applies more generally when wages vary at all across the two states, since announcing the lower wage state always maximizes ex post profits. Thus the only incentive compatible contract has a constant wage.

[31] For ease of presentation, the proposition considers the case where there is a single period-2 state $\hat{x}$ in the initial situation. If there are other states in which the no-undercutting condition is not binding, the argument can be extended straightforwardly. The argument also extends readily to non-equi-probability perturbations.

[32] Formal proof is provided in Online Appendix B.3.

[33] Using the same calibrations as in Footnote 18, we find that the standard deviation of unemployment in the rigid wage region is increased by approximately 60% relative to the restricted model model. The proposition will also hold *mutatis mutandis* in the unrestricted

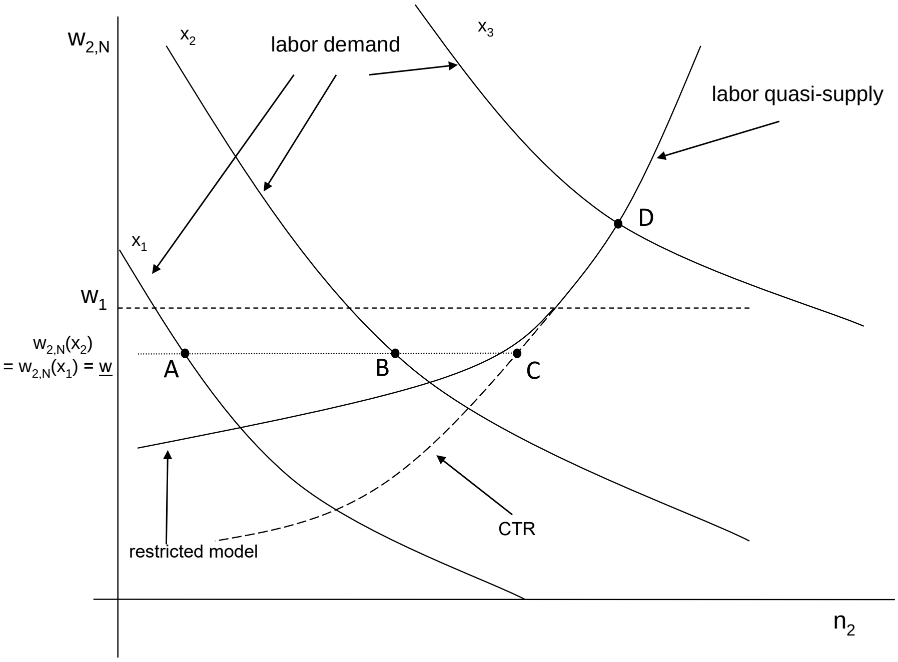

Figure 4: A rigid wage under asymmetric information

This argument works for a small difference in the two shocks; however, for a very wide variation in shocks, the lower $w_{2,N}$ in the restricted model symmetric information equilibrium might be so low — below the optimal level in the other state — that switching to it reduces profits from new hires. This fall in profits is unlikely to outweigh the gains from cutting $w_{2,I}$ though, as the latter are first-order and large, while around the optimal hiring wage the change in profits on cutting $w_{2,N}$ will be second-order.[34] The same forces exists when there are multiple states implying a wage floor $\underline{w}$ to which wages of both new hires and incumbents are equal across a wide range of states. An incentive compatible contract is illustrated by points A and B in Figure 4, assuming there are only two states, x_1 and x_2.[35]

model provided the no-undercutting condition holds strictly at $\hat{x}$.

[34]For very high rates of turnover (such that incumbents become a very small fraction of the workforce) and for large negative shocks such that wages are not very close together in the restricted model solution, the latter solution *will* satisfy incentive compatibility. However, in our simulations with parameterizations as in Footnote 18, constant wage contracts remain optimal across negative shocks, where the worst shock is up to 50% below the best shock, even when the turnover rate is as high as 80%. For lower turnover rates, the range of shocks where constant wages are optimal is still higher.

[35]The level of the wage floor will depend on the severity of the distribution where the constrained regime applies as, roughly speaking, the wage floor averages across the wages on

Part (iii) says that in the state with the highest $w_{2,I}$, if the no-undercutting condition is not binding, new-hire wages are at the CTR solution. Intuitively, continuing the previous discussion, suppose that there is a third state $x_3 > x_1, x_2$, such that $w_{2,N}(x_3) = w_{2,I}(x_3) = \underline{w}$; now suppose that this state of nature improves (i.e., consider perturbing the model by increasing x_3 holding all else constant). As x_3 increases, the new-hire wage that is optimal in the CTR model, $w_{2,N}^{CTR}(x_3, w_1, n_1)$, that is, ignoring the no-undercutting and incentive compatibility constraints in that state, rises above $\underline{w}$. This happens when the demand and CTR quasi-supply curves intersect above point C in Figure 4. It is incentive compatible to have $w_{2,N}(x_3)$ at the optimal level (see point D) with $w_{2,I}(x_3) = \underline{w}$: announcing a lower state from state x_3 will reduce profits ($w_{2,N}$ will be at a suboptimal level, while $w_{2,I}$ will be the same). In fact, the firm can do even better: $w_{2,I}$ will be slightly higher than $\underline{w}$.[36] For sufficiently favorable x_3, $w_{2,I}$ can increase all the way to w_1 without violating incentive compatibility, but as shown in general in (iv), it is never optimal to exceed w_1. Nevertheless, due to the incentive constraints incumbent wages are procyclical — though within the restricted interval of wages $[\underline{w}, w_1]$ — even over a range of "positive" shocks (i.e., such that $w_{2,N}^{CTR}(x, w_1, n_1) > w_1$) in contrast to the symmetric information case, something that may accord better with empirical evidence.[37]

When there is just one state in which wages exceed a wage floor, the latter logic also implies that the restricted model quasi-supply curve under asymmetric information coincides with the CTR one for a range of wages below w_1, down to the "wage floor" $\underline{w}$ (in contrast to the symmetric information case).[38] Therefore,

the restricted model quasi-supply curve in this region. In the empirical section, we proxy for productivity in this region with forecast unemployment conditional on the latter being above its long run mean.

[36]There will now be a cost of deviating by announcing a lower state, given that the new-hire wage will fall below the optimal level, so $w_{2,I}(x_3)$ can increase towards w_1, increasing incumbent wage costs by a corresponding amount (recall that moving $w_{2,I}$ towards w_1 will improve ex ante profits). Hence, $w_{2,I}(x_3)$ will be set to exactly satisfy the incentive compatibility constraint subject to not exceeding w_1. Initially, this scenario is a comparison between a second-order cost and a first-order gain, so the increase in $w_{2,I}$ is itself second-order to avoid violating the incentive constraints.

[37]In the simulations, incumbent wages increase up to the point where the new-hire wage is approximately 10% higher than w_1.

[38]If there are multiple states with wages above the wage floor, we can establish the following result (details available on request). For any equilibrium satisfying monotonicity in the sense that whenever $w_{2,I}(x) > w_{2,I}(x')$, $Z_2(x) \geq Z_2(x')$ and $w_{2,N}(x) \geq w_{2,N}(x')$, and also no undercutting binds in x if and only if $w_{2,I}(x)$ is below some critical $w_{2,I}$ (which can be the empty set), then only downward incentive compatibility constraints can bind, and for all states x where no undercutting is not binding, $w_{2,N}(x) \leq w^{**}(x)$. That is, new-hire wages are no higher than the CTR level. Moreover, if only local downward constraints bind (as is true in our

the region of "flexibility" for new-hire wages extends further (i.e., wages are initially more flexible downwards, but then fully rigid) than in the symmetric information case. Consider point C in Figure 4: if there is a state with demand curve passing through this point, the fact that incentive compatibility lowers the incumbent wage even in such a state implies that the no-undercutting condition first binds only at lower levels of the new-hire wage so that $w_{2,N}$ will be set at this level.

Discussion

It is useful to contrast our result with earlier models in the asymmetric information implicit contracting literature, such as Grossman and Hart (1981). A firm employs risk-averse workers with a decreasing returns to scale production function, as here, and likewise with asymmetric information where the firm knows the state. If the firm is risk neutral, then the first-best contract can be implemented, but if the firm is risk averse, it would prefer to lay-off some workers in some productivity states where it would be efficient to employ them (in that their marginal products exceed their reservation wage). The idea is that if a firm fully insures workers (i.e., across states and whether workers are employed or not) then to implement efficiency the difference between the wage for an employed worker and what an unemployed worker receives, must equal the reservation wage. This will induce the firm to employ up to the point where the reservation wage equals the marginal product, as in the first best. However the firm would bear all the risk; a risk averse firm would optimally set the contract to shift some risk to workers, and to implement this under asymmetric information the above difference must exceed the reservation wage, shifting some risk from the firm to workers when productivity is low. But this implies the firm will employ fewer workers than is efficient in some states.

This model differs from the current one, aside from having a risk averse firm, in that it is effectively a one-period setting in which a firm has a pool of workers associated with it with which it contracts (the firm and workers enter into a contract before the state is known, but workers may be immobile once contracted). Not all workers need be employed in all states, although the firm can insure those who do not have a job. In our case, by contrast, the employment decision

simulations) and $w_{2,I} < w_1$ for higher states (higher by $w_{2,I}$ ranking), it is a strict inequality: $w_{2,N}(x) < w^{**}(x)$. The intuition here is that cutting $w_{2,N}(x)$ a small amount below $w^{**}(x)$ imposes only a second-order cost in state x, but announcing x in a higher state will suffer a first-order cost by this change; this cut would relax the incentive compatibility constraint and permit a higher $w_{2,I}(x')$.

is a job opening/hiring one rather than an employment/layoff one; moreover the insurance of workers concerns the period 2 wage of incumbents hired in period 1.[39]

Our base assumption is that firm employment is unobservable to workers or not contractible, as in, e.g., Grossman and Hart (1983).[40] This contrasts with work such as Chari (1983) and Green and Kahn (1983). In practice, however, the level of employment in a firm can be difficult to define precisely. For example, if the relevant employment level is at the plant, the firm may be able to move production to other plants within the same company, making it difficult to condition on employment (as argued by Stiglitz, 1986). We also consider an extension in an online appendix in which we allow contracts to depend on employment levels, and we show that (when shocks are not too far apart) a similar logic applies locally and that wages are essentially constant.

Of course if the aggregate state is contractible in some way, then the asymmetric information problem would be resolved. Outside conditions, such as labor market tightness or the value of Z_2, may be difficult to contract over; in the equilibrium the wage itself is not informative as it is constant across the bad states. An approach adopted in the literature which implies that asymmetric information may nonetheless affect output when the aggregate state is observable, is to assume that the distribution of idiosyncratic shocks *depends* on the aggregate shock. Extending Grossman and Hart (1981), Grossman et al. (1983) use the idea is that there may be aggregate states where there is no asymmetric information, and employment is efficient, and other states where firms are subject to asymmetric shocks which are not known to workers at those firms. Because, in the latter aggregate state, employment is inefficiently low on average due to the logic explained above, unemployment varies with the aggregate state.

Consider a variant of our model in which there are multiple independent sectors but only aggregate unemployment is observable, and suppose that sectoral productivity in period 1 is an aggregate plus an i.i.d. sectoral shock. In high aggregate productivity states most sectors receive positive shocks relative to period 1, and so will be paying new-hire wages above the incumbent wage. In

[39]If our model were one-period, or equivalently, if turnover was 100%, then the restricted model quasi-supply curve coincides with the CTR one, and there is no incentive to deviate from the optimal symmetric information contract to benefit from savings on incumbent wages, so the optimal contract is implementable.

[40]Grossman and Hart (1983) consider a single worker model in which a worker is either employed or unemployed, or equivalently, a firm with many workers but where the level of employment is, as here, not contractible.

such states there is no issue with asymmetric information. In the low productivity states, sectors will mostly have low productivities relative to period 1. Workers will be able to deduce from aggregate unemployment what the aggregate state is, and so if unemployment contingent contracts are feasible, worse aggregate states will be associated with lower wages. However, across negative shock sectors in a particular aggregate state, wages will be the same.

5 Testing the Model's Predictions

In this section, we present tests of the salient features of our model. Our main focus is on the predictions of the equilibrium in the restricted model under asymmetric information (henceforth RAI) as laid out in Proposition 3. However we are also able to examine how this model does against the versions where undercutting occurs and the extent to which asymmetric information is important. We use panel data from the IAB Beschäftigten-Historik to extract composition free estimates of the annual aggregate wages of new hires and incumbents in Germany from 1978 to 2014.

Referring to high and low productivity states as up- and downswing periods, respectively, the RAI model implies three broad stylized facts about new-hire and incumbent wages.

Implication A: In downswings new-hire and incumbent wages are equal and relatively sticky.

Implication B: Wages in downswings are more closely related to forecasted economic conditions rather than ex post current economic conditions (in particular, they are better related to the forecast of productivity in downswings rather than productivity itself).

Implication C: In upswings the response of incumbent wages is damped relative to those of new hires, with the latter fully adjusting to current conditions.

Implication B is the most important of the three. It relates directly to the main innovation of this paper — the analysis of wage contracts under asymmetric information — and implies distinctive wage behaviour which to our knowledge is novel. If asymmetric information was unimportant, i.e., if workers were fully informed about the current state, then regardless of undercutting, Implication B would not hold; we would find instead that ex post economic conditions in downswings matter more than their forecasts.[41]

[41] Recall from Proposition 1 that the wage would lie at the intersection of the labor quasi-

Implication A relates directly to undercutting. If undercutting *does* occur then implication A would fail; in downswings incumbent wages would be rigid while new-hire wages would fall (Proposition 2). We are able to test for this too.

For Implication C see the discussion of part (iii) of Proposition 3; that incumbent wages increase to a limited extent in upswings, in contrast to the symmetric information model in which they are constant.[42]

In our empirics we adopt the traditional approach of using (demeaned) unemployment as an indicator of the aggregate state rather than productivity itself. Following the empirical literature in this area we define upswing (downswing) years as those with below (above) average unemployment. We execute two analyses: one using a single "aggregate" wage series and another where we examine the behaviour of wages in each of six broad sectors[43] of the economy. To the extent that each sector approximates a segregated labour market then drilling down to sector level will offer greater power to our tests, as we argue below.

5.1 The Data

For our empirical exercises, we use the IAB Beschäftigten-Historik (BeH, version 10.01), the Employee History File of the Institute for Employment Research (IAB) of the German Federal Employment Agency. The BeH covers all workers who were at least once employed subject to social security in Germany since 1975. Not covered are self-employed, civil servants (Beamte), family workers assisting in the operation of a family business, and regular students. The BeH includes roughly 80% of the German workforce. To protect data privacy, we are not allowed to work with the universality of the BeH. Therefore, we use a 20% random sample of all workers that worked full-time during at least one year since 1975.[44]

The BeH is organized by employment spells. A *spell* is a continuous period

supply and demand curves, so the wage response would be muted relative to upswings but depend on ex post demand.

[42] At the other extreme, in a competitive model where workers can costlessly move to higher paying firms incumbent wages would move with new-hire wages in upswings.

[43] 1) Mining, Agriculture, etc., 2) Manufacturing, 3) Power, 4) Construction, 5) Retail, and 6) all other activities. Please refer to Table C.3 in Online Appendix C for more detailed information.

[44] More precisely, we focus on "regular workers" according to the definition used in the Administrative Wage and Labor Market Flow Panel (see Stüber and Seth, 2018): a regular worker is employed full time and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots). Therefore, all (marginal) part-time employees, employees in partial retirement, interns, etc., are not considered regular workers.

of employment within an establishment in a particular calendar year. Hence, the maximum spell length is 366 days. For each identified full-time worker of our sample, we observe all existing employment spells — including part-time employment, apprenticeships, etc. These spells are needed to clearly identify new-hire spells.

We define a new-hire spell as a worker's first spell at the establishment.[45] Hence, a worker's tenure in an establishment that spans more than one calendar year will consist of multiple spells, with the first being classified as a new-hire spell. For new hires we focus exclusively on workers transitioning to employment from unemployment in our analyses, for reasons we explain in Section 5.2. We define these hires as workers who were without a job for over four weeks before arriving at the firm.

Our dependent variable is the real average daily wage of a worker over any spell. As the earnings data are right-censored at the contribution assessment ceiling ("Beitragsbemessungsgrenze"), only non-censored wage spells are considered in the analyses.[46] To calculate the average daily real wage and real output per capita in 2010 prices, we use the German Consumer Price Index (CPI). As a proxy for the state of the business cycle (aggregate productivity in our model), we follow the literature (e.g., Bils, 1985; Solon et al., 1994), and use the demeaned aggregate unemployment rate, which we obtained from the Federal Unemployment Agency. The CPI and unemployment series are displayed in Table C.1 in Online Appendix C.

For our analyses, we restrict our attention to employment spells of full-time workers[47] aged 16 to 65 years from West Germany for the period from 1978

[45]Re-hires are therefore not identified as new hires. Our decision to treat returning workers as incumbents is because of the relatively short time of absence; 70% of returners returned after an absence of less than one year, and returners' average length of time away is approximately 20 months. This suggests that these spells are for workers who have long-term relationships with the establishment and whose absences were temporary (for reasons such as paternity/maternity leave).

[46]We drop spells with wages ≥ 0.98 * the contribution assessment ceiling. Dropping top-coded spells leads to an under-representation of highly qualified workers, making the results somewhat less generalizable. Because the wages of highly qualified workers are less likely to be covered by a collective bargaining agreement (see, e.g., Düll, 2013) and because uncovered wages are more flexible than covered wages (see, e.g., Devereux and Hart, 2006), we likely slightly underestimate the wage cyclicality. For a quantitative evaluation of the effect of dropping censored spells, see, e.g., Appendix A of Stüber and Beissinger (2012).

[47]The BeH documents only total spell earnings, not hours worked in that spell. We therefore consider only full-time workers, as these workers' hours are likely to be acyclical. In earlier work that is available upon request, we analyse the time series properties of an extraneous estimate of the average hours worked in a year by full-time employees in Germany. We find cyclicality — in the sense of having a significant correlation with output — to be relatively

to 2014. We do not use the first few years of the dataset, as we use workers' establishment tenure as an independent variable in our analyses.[48] We further keep employment spells only if the workers are employed on December 31^{st} of the respective year.[49]

The final dataset used in our analyses contains over 97.8 million employment spells for nearly 9 million workers working for more than 2.8 million establishments (see Table C.2 in Online Appendix C). The BeH contains an establishment identifier, but henceforth, we refer to establishments as "firms" in keeping with the phrasing used in the discussion of the theory.[50]

5.2 Extracting Composition-Bias-Free Estimates of New-Hire Wages

We wish to test the model's predictions concerning the cyclical behavior of new-hire wages relative to those of incumbents. To do this, one must extract estimates of these wages from the panel data, controlling for composition bias. Following Solon et al. (1994), this can be achieved with a two-step method. In the first stage, year effects are extracted from the panel using year dummies while controlling for worker-firm characteristics. In the second stage, the year effects are treated as composition-controlled estimates of the average new-hire wage in each year. In the two-period asymmetric information model, new hires come from unemployment, not from other firms. Hence, the wage year effects that we would like to identify are those for new hires arriving directly from unemployment — so called UE transitions. Gertler et al. (2020) argue that the quality composition of UE transitions is acyclical because they eliminate the job ladder moves initiated by on-the-job search which are widely believed to be the source of procyclical match quality. We would agree that UE transitions are free of job ladder movers in the traditional sense but we would not agree that the quality of UE transi-

weak.

[48]We drop all spells for which we cannot calculate establishment tenure, i.e. spells that started on (or before) January 1^{st}, 1975.

[49]This specification implies that we only ever have a maximum of one spell per worker per year, so when we compute yearly averages over spells, we do not more heavily weight those workers with multiple within-year spells. It also excludes most short-lived spells in the data, particularly temporary summer work.

[50]The main results of this paper hinge on estimates that control for match fixed effects, with the underlying assumption being that matches are with establishments, not firms. However, even if matches are formed at the firm level, then using worker-establishment fixed effects will absorb them in any event; their use in this case may be inefficient but will not bias the estimated year effects.

tions are necessarily acyclical. Using CPS data, Mueller (2017) argues that the quality of the unemployed pool is countercyclical. Taking this as a stylised fact, both pro- and countercyclical match quality are conceivable. For example, it maybe that in upswings when the number of vacancies is growing, a (imperfect) screening process of applicants for jobs results in higher quality workers being over represented in UE transitions and UE match quality would be procyclical. Alternatively if matching out of unemployment was random then UE match quality may be countercyclical. In the former (latter) case estimates of new-hire wage cyclicality would be biased away from (towards) zero. To avoid these issues we use *match fixed effects* (MFE) to control for worker quality but this too is not without problems. If the amount by which new-hire wages are above/below that of incumbents (henceforth new-hire "premia") and if these premia are permanent during the workers' tenure with the firm then MFE will absorb them and the new-hire premia estimate will be centred on zero. By contrast if these premia are temporary then they will show up in the estimates — at least to some extent.[51] Our view is that it is highly unlikely that new-hire premia will be fully persistent; for one thing, on the job search will limit the time which a newly hired worker may be paid below his marginal product. In fact a number of papers have argued along these lines, most notably Hagedorn and Manovskii (2013). In any event and in extremis even if MFE did eradicate new-hire premia entirely this would be a double edged sword in terms of support for our theory; one of the theory's key predictions is that there is a new-hire premium in upswings and if the cyclical new-hire premia are in fact permanent then using MFE would result in us finding no support for this prediction of our model. In this respect, at least, the use of MFE is conservative because it works against finding in favour of our model.

In the first stage, the primary specification to be estimated is the panel regression

$$w_{ijt} = m_{ijt} + \sum_{\tau=1}^{T} \beta_{\tau}^{I} I_t^{\tau} + \sum_{\tau=1}^{T} \beta_{\tau}^{E} E_t^{\tau} + \sum_{\tau=1}^{T} \beta_{\tau}^{N} N_t^{\tau} + \sum_{k=1}^{2} \lambda_k age_{it}^{k} + \sum_{k=1}^{4} \phi_k ten_{ijt}^{k} + v_{ijt}, \tag{10}$$

where w_{ijt} is the log of the real average daily wages of worker i in firm j during

[51]It is easy to show that using MFE will cause downward bias to new-hire premia but whilst this may affect small sample power of a significance (from zero) test it will not drive the estimate to zero asymptotically. We return to this issue when we present our empirical estimates below.

year t, and v_{ijt} is an error term.

The equation allows for three distinct sets of year effects written in the first three summation terms. The first consists of the dummies I_t^τ ($\tau = 1, \ldots, 37$) with coefficients β_τ^I where I_t^τ equals one if $t = \tau$ and the worker is an incumbent, but is zero otherwise. An incumbent is currently defined as a worker with more than 365 days of tenure. Later we consider an alternative definition in a robustness analysis. The β_t^I coefficients are the incumbents' year effects. The second and third set of dummies E_t^τ and N_t^τ take the value of one if the wage is from an EE new hire or an UE new hire, respectively.[52] Otherwise, $t = \tau$ is equal to zero. The β_t^E and β_t^N are the corresponding year effects. In the further analyses we focus on the year effects of incumbents (β_t^I) and UE new hires (β_t^N). The variable age_{it} is the worker's age in years, and ten_{ijt} is the worker's firm tenure measured in days at the end of the spell. Finally, m_{ijt} is a MFE. Note that MFE's control for the sum of a firm j's effect plus a worker i's effect plus a match quality effect.

5.3 Testing the Model

5.3.1 Tests Based on Correlations of Wages with Unemployment

Here we examine the empirical support for the model's three implications outlined in Section 5 above.

As noted — and as is now standard in this literature — we use the unemployment rate as a proxy for the state of the business cycle (i.e., a proxy for the model's aggregate productivity). We categorise the data into up- and downswing years according to whether demeaned unemployment is, respectively, negative or positive.[53] We start with a traditional exercise of examining the comovement between unemployment and wages over the business cycle for new hires and incumbents, extended to allow for asymmetric responses in upswings and downswings. Explicitly we consider

$$\beta_t^i = \gamma_d^i \widetilde{u}_{dt} + \gamma_u^i \widetilde{u}_{ut} + \varepsilon_t \quad i = N, I, \tag{11}$$

where, denoting the demeaned unemployment rate by $\widetilde{u}_t$, $\widetilde{u}_{ut}$ ($\widetilde{u}_{dt}$) equals $\widetilde{u}_t$ when $\widetilde{u}_t < 0$ (> 0) and is zero otherwise. Superscripts N and I denotes new hires from unemployment and incumbents, respectively. In keeping with the literature

[52] We count all transitions into employment that are not EE transitions as UE transitions. Hence our UE transitions also include transitions from non-employment into employment.

[53] Defining downswings (upswings) as years when unemployment is above (below) its full sample mean gives us 15 upswing years and 22 downswing years (see Table C.1).

we refer to the wage cyclicality coefficients (the γ's) as semi-elasticities. We should emphasise at this point that the γ's are not structural parameters but are merely the (normalised) sample covariances between wages and unemployment in upswings and downswings respectively.

First-differencing (11)[54] gives

$$\Delta\beta_t^i = \gamma_d^i \Delta\widetilde{u}_{dt} + \gamma_u^i \Delta\widetilde{u}_{ut} + \Delta\varepsilon_t \quad i = N, I. \tag{12}$$

We estimate (12) using composition controlled wages from (10). The results for new hires from unemployment and incumbents are given in the second column of Table 1 below with t-ratios (which here and throughout the paper are computed using standard errors that are robust with respect to heteroscedasticity and first-order error autocorrelation) given in brackets.

Table 1: Estimates of Upswing and Downswing Semi-Elasticities

γ_u^N	γ_u^I	γ_d^N	γ_d^I	γ_{fd}^N	γ_{fd}^I	$\gamma_u^N - \gamma_u^I$	$\gamma_{fd}^N - \gamma_{fd}^I$
−1.300	−0.985	−0.618	−0.507	−0.586	−0.525	−0.315	−0.061
(4.90)	(3.35)	(1.76)	(1.62)	(2.51)	(1.99)	(2.96)	(0.47)

Note: γ_u^i (γ_d^i) is semi-elasticity in upswings (downswings) of new hires from unemployment ($i = N$) and incumbents ($i = I$), respectively. Subscript f indicates the use of forecasted unemployment instead of actual unemployment for downswings.

Both, new-hire and incumbent upswing semi-elasticities are highly significant and correctly signed. The downswing estimates are small relative to their upswing counterparts and are roughly the same for new hires and incumbents. Their significance is borderline. Taken together, these outcomes are supportive of implication A (in downswings new-hire and incumbent wages are equal and relatively sticky). However the large and significant upswing semi-elasticity for incumbents jars somewhat with implication C; the model predicts a muted response of incumbent wages to upswings. Nevertheless, the new-hire upswing elasticity is larger than that for incumbents and significantly so, as the penultimate column indicates. It is possible that some degree of on the job search — not allowed for in our model — is driving the incumbent semi-elasticity upwards.

[54]This is now standard procedure to avoid inference issues associated with the unit root in wages (e.g., Gertler et al., 2020). However Gertler et al. (2020) first difference individual wages in a one-step panel estimation whereas we first difference wage effects ($\beta' s$) derived from the panel at the second stage. Unlike Gertler et al. (2020) therefore, our first differences do not eliminate fixed effects; this we do in the first stage.

Overall the results appear to offer some support for implication C above.

We now turn to examine implication B — in downswings wages for both classes of workers are more closely related to the forecast of the state of the economy ($\widehat{x}$ in the theory) rather than its actual state. To examine this, we estimate a simple forecasting model for unemployment in "bad" states. Explicitly we estimate an AR(2) model for unemployment using only those years in which unemployment was above its long-term mean ($\widetilde{u}_t > 0$). We denote this forecast as $\widetilde{u}^f_{dt}$. If we call the forecast error for these downswing years e_{dt} then we have

$$\begin{aligned} \widetilde{u}_{dt} &= \widetilde{u}^f_{dt} + e_{dt} \qquad \text{when } \widetilde{u}_t > 0 \\ \text{and where } \widetilde{u}^f_{dt} &= 0 \qquad \text{when } \widetilde{u}_t < 0 \end{aligned}$$

Implication B says that in downswings wages of incumbents and new hires should respond to $\widetilde{u}^f_{dt}$ rather than $\widetilde{u}_{dt}$. To test this we amend (11) to give

$$\beta^i_t = \gamma^i_{fd}\widetilde{u}^f_{dt} + \gamma^i_{fu}\widetilde{u}_{ut} + \varepsilon_t \quad i = N, I.$$

Once again we first difference to get

$$\Delta\beta^i_t = \gamma^i_{fd}Dx_{dt} + \gamma^i_{fu}Dx_{ut} + \Delta\varepsilon_t \quad i = N, I \tag{13}$$

where

$$Dx_{ut} = \begin{cases} 0 \text{ if } \widetilde{u}_t > 0 \\ \widetilde{u}_t - \widetilde{u}^f_{dt-1} \text{ if } \widetilde{u}_t < 0 \;\&\; \widetilde{u}_{t-1} > 0 \\ \widetilde{u}_t - \widetilde{u}_{t-1} \text{ if } \widetilde{u}_t < 0 \;\&\; \widetilde{u}_{t-1} < 0 \end{cases},$$

$$Dx_{dt} = \begin{cases} 0 \text{ if } \widetilde{u}_t < 0 \\ \widetilde{u}^f_{dt} - \widetilde{u}^f_{dt-1} \text{ if } \widetilde{u}_t > 0 \;\&\; \widetilde{u}_{t-1} > 0 \\ \widetilde{u}^f_{dt} - \widetilde{u}_{t-1} \text{ if } \widetilde{u}_t > 0 \;\&\; \widetilde{u}_{t-1} < 0 \end{cases}.$$

We estimate (13) and compare the t-ratios of the downswing semi-elasticities with those from (12). If forecasted rather than actual unemployment is the relevant correlate of wages in downswings then the significance of the downswing γ's would increase and this would be a finding in favour of asymmetric information.[55]

[55] It is tempting to say that in this scenario the actual value is subject to classical measurement error and that the estimates should be downward biased. However the first differencing implies that this is not the case; the "measurement error" is correlated with the measure in

Before proceeding we note that the AR(2) coefficients for the downswing unemployment forecast are highly significant (the p-value is less than 0.0001) despite the scarcity of data points; unemployment has clearly defined dynamic momentum in the annual frequency during downswings.

The results for γ^i_{fd} are also given in Table 1.[56] We see that for both sets of workers the unemployment forecast in downswings is more significant than its actual value. Unsurprisingly, $\gamma^N_{fd} - \gamma^I_{fd}$ is insignificant. The estimates of γ^i_{fu} and their t-ratios were virtually identical to those for γ^i_u and so we do not report them.

Before we close this section we return to our choice of using demeaned unemployment as our cyclical indicator. As noted, this choice was inspired partly by our model but also because it is a standard choice of cyclical indicator in this context. However and unlike much of the other papers in this literature, our results also hinge crucially on how we split the sample into up- and downswings. In the absence of a more formal mechanism to effect this split (such as a model for structural unemployment) it would be comforting to obtain some kind of external validation for our classification of up- and downswing years. To do so we examine estimates of Germany's output gap produced by the International Monetary Fund (IMF, see De Masi, 1997). The correlation of demeaned unemployment with this series is -0.6. More importantly in 28 of the 35 years[57] of the sample the two series "agree " on the classification of data into up- and downswing years. In seven of the eight years where there is a conflict in this classification demeaned unemployment is very close to zero. Finally in an earlier empirical paper again using wage data from the BeH (see Snell et al., 2018) we tested for equal treatment in up- and downswings. An important difference there was that we defined upswings (downswings) as periods of positive (negative) GDP growth rather than above (below) average unemployment. In that context we found evidence of equal treatment in both up- and downswings but in the case of the former the evidence was marginal. Overall then, despite the different cyclical measure, the previous paper also offers some support for implications A and C.

We summarise the results so far. We find strong support for implications A and B above of the RAI model and partial support for implication C. By contrast

the first-difference estimation.

[56]Note that the forecast is a generated regressor. However the size of regular significance tests is not affected due to the null of zero (Pagan, 1984). The power of these tests may be affected however, especially if attenuation bias results. Note also we do not report γ^i_u again — they are the same as before due to orthogonality of the regressors.

[57]The IMF series begins in 1980 so we lose one data point relative t our core sample.

there is little support for the symmetric information version of the model. The fact that there appears to be equal treatment in downswings is evidence against the undercutting equilibrium in the model.

5.3.2 Tests Based on Sectoral Data

Empirical exercises such as the one in the previous section now abound in the literature; a large panel data set on wages is used to synthesise composition free estimates of aggregate wages in order to ascertain (some aspect of) the cyclicality of the economy's wage. Despite the huge dimension of the panel data from which the annual aggregate is derived the fact remains that the results rest on a small number of time series observations — equivalently put there are rarely more than a handful of business cycles on offer from which to draw inferences.[58] Here we try and bring more data to bear on our empirics by drilling down to the sector level. Explicitly, we obtain composition corrected wages for new hires and incumbents as we did above but now for six broad economic sectors. These are 1) Mining, Agriculture, etc., 2) Manufacturing, 3) Power, 4) Construction, 5) Retail, and 6) all other activities.[59] If each sector approximates a segregated labour market and if we were to obtain measures of sectoral unemployment rates then we could repeat our estimation exercises sector by sector and see if the predictions of our model hold up in each case. Unfortunately, data for sectoral unemployment rates do not exist. Instead we assume that each sector's unemployment rate u_{it} co-moves with the aggregate u_t — at least to some extent. In particular suppose that

$$u_{it} \approx \delta_i u_t + error_t \quad i = 1, \ldots, 6.$$

Repeating the above estimations for each sector separately would then yield semi-elasticities that were each scaled up by δ_i. Clearly model implications A, B, and C could be assessed in exactly the same way as before. If sectors were not segregated and our model was true, all of the estimates should be close to those obtained for the aggregate. If by contrast the sectors were segregated labour markets, then the potential heterogeneity in cyclical responses could add power to our tests. Put another way, if the aggregate estimates are masking some

[58] Alternatively — and as is now more the mode nowadays — one can estimate these cyclicalities directly in one step in the panel dimension as long as one uses appropriately clustered standard errors. Our point remains the same namely that only year to year variation in a single measure is being used.

[59] Please refer to Table C.3 in Online Appendix C for more detailed information.

sectoral cyclical responses that are at odds with our model, the new estimates would expose this.

We re-estimate (12) and (13) for each of the six sectors. The results — the analogues of those for the aggregate given in Table 1 — are given in Table 2.

Table 2: Sectoral Estimates of Upswing and Downswing Semi-Elasticities

Sector	1	2	3	4	5	6
γ_u^N	−1.122	−1.197	−1.359	−1.018	−1.072	−1.317
	(2.91)	(3.82)	(3.47)	(2.01)	(3.85)	(5.56)
γ_u^I	−0.568	−1.195	−0.646	−1.330	−0.687	−0.810
	(2.63)	(4.55)	(1.94)	(2.98)	(2.12)	(2.32)
γ_d^N	−0.613	−0.862	−0.290	−0.603	−0.773	−0.754
	(1.53)	(3.23)	(0.91)	(1.76)	(2.07)	(2.20)
γ_d^I	−0.654	−0.783	−0.002	−.0701	−0.662	−0.290
	(1.72)	(2.67)	(0.00)	(1.87)	(2.04)	(0.87)
γ_{fd}^N	−0.847	−0.720	−0.656	−0.634	−0.680	−0.786
	(2.68)	(3.33)	(1.96)	(1.97)	(2.04)	(3.72)
γ_{fd}^I	−0.800	−0.672	−0.306	−0.176	−0.450	−0.486
	(2.05)	(2.15)	(1.06)	(0.60)	(1.83)	(1.79)
$\gamma_u^N - \gamma_u^I$	−0.552	−0.002	−0.713	0.312	−0.385	−0.507
	(2.21)	(0.00)	(2.65)	(1.32)	(3.80)	(3.55)
$\gamma_{fd}^N - \gamma_{fd}^I$	0.046	−0.048	−0.351	−0.458	−0.230	−0.300
	(0.20)	(0.29)	(0.89)	(1.55)	(1.73)	(2.26)

Note: γ_u^i (γ_d^i) is semi-elasticity in upswings (downswings) of new hires from unemployment ($i = N$) and incumbents ($i = I$), respectively. Subscript f indicates the use of forecasted unemployment instead of actual unemployment for downswings.

In sectors 1, 3, 5, and 6 we find significantly higher upswing elasticities for new hires from unemployment than for incumbents. The amounts by which the new-hire upswing semi-elasticity exceed the incumbents' one in these sectors varies between 0.39 and 0.71 with an average of around 0.54.[60] However in sectors 2 (manufacturing) and 4 (retail) we do not find a significant difference in upswing elasticities. This is consistent with equal treatment in both up- and downswings — we return to this issue below. In all but sector 6 (all other activities) the downswing elasticities are not significantly different across the two types of workers. These findings offer broad support for implication A of the RAI

[60] Recall that semi-elasticities with respect to sectoral unemployment are unknown; the estimates here are a factor δ_i times these unknown values. However it is the significance of the difference in upswing semi-elasticities that is important for validating the model and for four of six sectors we find this.

version of the model (in downswings new-hire and incumbent wages are equal and relatively sticky). There is also broad support for implication B — that in downswings, forecasted unemployment is a better correlate of wages than actual — although less strong than for implication A. Forecasts are better correlates in eight out of the 12 cases and in a further two cases there is little to choose between the two variates. All 12 of the upswing semi-elasticities are robustly significant and ten of the 12 downswing ones are likewise. Finally note that here and in the aggregate results (see Table 1) the downswing semi-elasticities are similar for both incumbents and new hires and in most cases significant. These findings are at odds with the CTR model which predicts that in downswings real wages will be constant for incumbents but falling for new hires (see Section 3.2).

The fact that in most cases forecasts of unemployment perform as well or better than ex post unemployment may to some extent be due to the fact that unemployment lags swings in productivity by one or two quarters. To assess whether not spurious factors such as this could be at play we estimate a counterfactual model — one where the upswing unemployment variable in (12) is replaced with its forecasts[61]. We do this for the six sectors and for the aggregate. Neither the baseline nor the asymmetric models imply that forecasts are the correct wage correlate in upswings so we would expect forecasts to be less statistically important than ex post values. In these counterfactual regressions the upswing elasticities in the six sectors fall markedly in value and significance with eight of the 12 becoming wholly insignificant whilst the other four are borderline. We should note the results are equally stark for the aggregate case also.

Finally we return to the manufacturing and construction sectors, sectors which seem to display equal treatment in both up- and downswings. Support for the asymmetric information version of the model is weaker also for these two sectors. The equal treatment models of Snell and Thomas (2010) and Snell et al. (2018) — models without search frictions — appear more relevant to these sectors. We speculate that these sectors are dominated by high skilled workers with well defined jobs. It may be that getting workers in post is less costly than elsewhere. The wedge between new-hire and incumbent wages is driven mainly by search frictions (absent in Snell and Thomas (2010) hence the equal treatment in both up- and downswings there). So if these search frictions are relatively small we would expect something approaching equal treatment in upswings.

We summarise by saying that the comovement displayed between composi-

[61] As before we use an AR(2) model this time estimated from upswing years only and again we find its coefficients to be highly jointly significant with a p-value of 0.022.

tion controlled aggregate wages and unemployment offers broad support for the no-replacement equilibrium in the asymmetric information version of our model. This support is reinforced by sector level comovements. There we see a lot of heterogeneity and many estimates are quite different to their aggregate counterparts. However very few of these differences are in directions that undermine the model's predictions.

6 Concluding Comments

We have considered a simple frictional model of the labor market which has equilibrium wage contracts where incumbents are not undercut and displaced by new hires leading to a degree of downward wage rigidity for new hires. The rigidity arises from worker risk aversion and a desire to limit temporal wage variation for incumbent workers, which also transmits to new hires in downturns. Because period-two new-hire wages are allocational, the response of unemployment and job openings to negative shocks is amplified. The main theoretical contribution is an extension in which we show that the interplay with asymmetric information can substantially enhance downward wage rigidity and increase the responsiveness of unemployment and job openings to productivity shocks. We find that empirical results from the German BeH panel data are broadly supportive of the predictions of the asymmetric information version of the model; new-hire wages respond to upswings more aggressively than those of incumbents and in downswings both classes of wage are relatively sticky and respond more to forecasts of the downswing state than the actual state itself.

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Online Appendix

A Extension in Asymmetric Information Model to Employment-Contingent Contracts

The analysis in in Section 4 of the paper concerned the case in which no variables that are observable to both parties can be contracted upon. While in a model which features a frictional labor market, it is plausible to suppose that it may be difficult to condition contracts on aggregate labor market variables such as wages offered by other firms, employment at the firm in which the worker is employed may be a variable that could be conditioned upon. Intuitively, in a low-productivity state, employment could be specified to be inefficiently low to discourage the firm from underreporting productivity in better states to avail itself of lower wages, given that such inefficiency harms profits more in the better state. Here we consider how matters change if employment-contingent contracts are possible; for small variations in productivity, in fact, it does not affect the constant wage result.

Proposition A.1 ***(Employment-contingent contracts)*** *In the restricted asymmetric information model* where period-2 employment is contractible *and with a single period-2 productivity state $\hat{x}$, suppose that for given parameter values, there is a unique equilibrium and that the no-undercutting condition binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states, $x' = \hat{x} - \varepsilon$ and $x'' = \hat{x} + \varepsilon$ (i.e., with expected value $\hat{x}$), and assuming the differentiability of equilibrium values,*[62] *equilibrium period-2 wages are approximately* constant *across these states, provided that the perturbation ε is sufficiently small; formally,* $\lim_{\varepsilon \to 0_+} \left(w_{2,N}(x'') - w_{2,N}(x') \right) / 2\varepsilon = 0$.[63]

A rough intuition for this result is as follows: Given that for a small perturbation in both states x' and x'', the no-undercutting condition continues to bind, and wages for incumbents and new hires are equal. If in the lower-productivity state, wages are lower by more than a second-order amount, there will be, as earlier, a first-order incentive for the firm in x'' to announce x', as there is a benefit both in terms of lower wages for period-1 hires and in terms of reducing

[62]That is, assuming that Z_2 is a differentiable function of ε in a neighbourhood of 0.
[63]Formal proof is provided below in section B.4.

the hiring cost for new hires. To prevent this, hiring can be reduced in x', which would be costly in the state x'', but it must be reduced by a large amount, given that hiring is initially (in the unperturbed equilibrium) optimal; this cut in hiring will also impose *first-order* costs in x', swamping any benefit from the lower wages (which are second-order).

B Proofs

B.1 Proof of Proposition 1

Proof. We derive the necessary conditions by considering the following Lagrangian, assuming that there is an interior solution.

$$\begin{aligned}\mathcal{L} = &(f(\tilde{q}_1(V_1)\overline{n}_1) - w_1\tilde{q}_1(V_1)\overline{n}_1 - k\overline{n}_1)\\ &+ E_{x'}[(f((1-\delta)\tilde{q}_1(V_1)\overline{n}_1 + \tilde{q}(w_{2,N}, x')\overline{n}_2; x') - w_{2,I}(1-\delta)\tilde{q}_1(V_1)\overline{n}_1\\ &- w_{2,N}\tilde{q}(w_{2,N}, x')\overline{n}_2 - k\overline{n}_2] + E_{x'}[\lambda_{x'}(w_{2,N} - w_{2,I})],\end{aligned}$$

where $\tilde{q}_1(V_1)$ is defined analogously to $\tilde{q}(w_{2,N}, x)$, $\lambda_{x'}$ is the multiplier on the $w_{2,N} \geq w_{2,I}$ constraint in state x' and recall $V_1 = v(w_1) + E[\delta Z_2(x') + (1-\delta)v(w_{2,I}(x'))]$. This expression leads to the FOCs:

$$\tilde{q}_1' v'(w_1)\overline{n}_1(f'(n_1) - w_1 + E_{x'}[f'(n; x')(1-\delta) - w_{2,I}(x')(1-\delta)]) - \tilde{q}_1(V_1)\overline{n}_1 = 0 \tag{B.1}$$

$$f'(n; x)\tilde{q}(w_{2,N}, x) - w_{2,N}\tilde{q}(w_{2,N}, x) - k = 0 \tag{B.2}$$

$$f'(n; x)\tilde{q}'\overline{n}_2 - \tilde{q}(w_{2,N}, x)\overline{n}_2 - w_{2,N}\tilde{q}'\overline{n}_2 + \lambda_x = 0 \tag{B.3}$$

$$\begin{aligned}&\tilde{q}_1' v'(w_{2,I}(x))(1-\delta)\overline{n}_1(f'(n_1) - w_1 +\\ &E_{x'}[f'(n; x')(1-\delta) - w_{2,I}(x')(1-\delta)]) - \lambda_x - (1-\delta)\tilde{q}_1(V_1)\overline{n}_1 = 0\end{aligned} \tag{B.4}$$

together with the complementary slackness conditions. Note that (B.2) implies (8) in the text.

From (B.1) and (B.4),

$$\frac{v'(w_1)}{v'(w_{2,I})}\left(q_1+\frac{\lambda_x}{\overline{n}_1(1-\delta)}\right)=q_1. \tag{B.5}$$

Using this to eliminate λ_x in (B.3):

$$f'(n;x)\,\tilde{q}'\overline{n}_2-\tilde{q}(w_{2,N},x)\,\overline{n}_2-w_{2,N}\tilde{q}'\overline{n}_2+q_1\overline{n}_1(1-\delta)\left(\frac{v'(w_{2,I})}{v'(w_1)}-1\right)=0. \tag{B.6}$$

There are two cases:

A. If $\lambda_x=0$, then (B.5) $w_1=w_{2,I}$, and (B.6) implies (7) in the text, and hence, we get (9). We characterize points that satisfy (9). For clarity, we let $\tilde{w}_{2,1}$ and $\tilde{\theta}_2$ denote the individual firm's values. Then

$$\tilde{q}'=\frac{dq}{d\theta_2}\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2\text{ constant}}\,.$$

From (3),

$$\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2\text{ constant}}=-\frac{pv'(w_{2,N})}{\frac{dp}{d\theta_2}\left(v(w_{2,N})-v(b)\right)},$$

and differentiating $q=p\cdot\theta_2$ to eliminate $\frac{dp}{d\theta_2}$, we obtain

$$\tilde{q}'=-\frac{dq}{d\theta_2}\frac{p\theta_2 v'(w_{2,N})}{\left(\frac{dq}{d\theta_2}-p\right)\left(v(w_{2,N})-v(b)\right)}. \tag{B.7}$$

After rearrangement,

$$\frac{q^2}{\tilde{q}'}=q^2\frac{\left(1-\frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)}{\theta_2\frac{dq}{d\theta_2}}\frac{v(w_{2,N})-v(b)}{v'(w_{2,N})}.$$

From our assumption on q, q^2 is increasing in θ_2, and the second term in the product is also increasing in θ_2 by assumption (it is the inverse of $\frac{q(\theta)\epsilon_q(\theta)}{(1-\epsilon_q(\theta))}$) while the final term is increasing in $w_{2,N}$. Thus, the locus of values of θ_2 and $w_{2,N}$ such that (9) holds is negatively sloped. Recall that $n_2=p(\theta_2)\,S_2$, and as $p'<0$, there is a one-to-one negative relationship between n_2 and θ_2. Therefore, (9) can be solved to give a positively sloped locus of values for n_2 and $w_{2,N}$ that is compatible with equilibrium.

Next, (B.2) is negatively sloped in $n_2-w_{2,N}$ space by $f''<0$ and $q(\theta_2)=q\left(p^{-1}(n_2/S_2)\right)$, $q'>0$, $p'<0$. Therefore, $(w_{2,N},n_2)$ is at the unique intersection point, denoted by $\left(w_{2,N}^{CTR}(x; w_1, n_1), n_2^{CTR}(x; w_1, n_1)\right)$ in the text. Since $w_{2,N} \geq w_1$ implies $\lambda_x = 0$ (see next line), claim (b) is established.

B. If $\lambda_x > 0$, then $w_{2,I} = w_{2,N}$ and from (B.5) $w_1 > w_{2,I} = w_{2,N}$, and (B.6) implies

$$(1-\delta)n_1 - \left(f'(n; x)\,\tilde{q}'\overline{n}_2 - w_{2,N}\tilde{q}'\overline{n}_2 - q\overline{n}_2\right) = n_1\left(1/v'(w_1)\right)\left((1-\delta)\,v'(w_{2,N})\right). \tag{B.8}$$

(This equation also follows from differentiating (5) with respect to $w_{2,N}$ after setting $w_{2,N} = w_{2,I}$.) Thus, eliminating f' using (B.2), and using $n_2 = q\overline{n}_2$,

$$1 + \frac{\left(1 - k\tilde{q}'/q^2\right) n_2}{n_1(1-\delta)} = \frac{v'(w_{2,N})}{v'(w_1)}, \tag{B.9}$$

so that as $w_{2,N} < w_1$, $k\tilde{q}'/q^2 < 1$, i.e., $k < q^2/\tilde{q}'$. (The locus of points satisfying (B.9) is the quasi-supply curve below w_1.) Holding n_2 (and hence θ_2) constant, $q^2/\tilde{q}'$ is increasing in $w_{2,N}$, so the locus of points $(n_2, w_{2,N})$ satisfying (B.9) must lie above — $w_{2,N}$ is higher — that defined by (9). At $w_{2,N} = w_1$ we have $k\tilde{q}'/q^2 = 1$, so the two loci coincide. Thus, the downward sloping (B.2) must intersect (B.9) at a higher wage and a lower value for n_2 than it would intersect (9). Thus, claim (a) is established.

Since $\lambda_x > 0$ if and only if $w_{2,N} < w_1$, the final claim of the proposition follows. ■

B.2 Proof of Proposition 2

Proof. If replacement occurs, as in Section 3.2, the firm must locally maximize profits plus weighted incumbent utility:

$$\begin{aligned} &f\left((1-\delta)n_1 + n_2; x\right) - w_{2,I}(1-\delta)(1-q)n_1 - w_{2,N}\left(q(1-\delta)n_1 + n_2\right) - k\overline{n}_2 \\ &\qquad + n_1\left(1/v'(w_1)\right)\left((1-\delta)(1-q)\,v(w_{2,I}) + \delta Z_2 + (1-\delta)\,qv(b)\right), \end{aligned}$$

where $\overline{n}_2$ is again the number of *new* jobs created, and $n_2 = q\left(\theta\left(w_{2,N}, Z_2(x)\right)\right)\overline{n}_2$. This situation differs from (5) in that the probability of replacement q is accounted for in the composition of period-2 workers and workers' period-1 utility. Then, differentiating with respect to $w_{2,I}$,

$$(1-\delta)(1-q)n_1 = n_1\left(1/v'\left(w_1\right)\right)\left(\left(1-\delta\right)\left(1-q\right)v'\left(w_{2,I}\right)\right),$$

so that $w_1 = w_{2,I}$, as expected. Differentiating with respect to $w_{2,N}$, we obtain

$$f'\left(n;x\right)\tilde{q}'\overline{n}_2 + (1-\delta)n_1(w_{2,I}-w_{2,N})\tilde{q}' - w_{2,N}\tilde{q}'\overline{n}_2 - q\left((1-\delta)n_1+\overline{n}_2\right) + \\ n_1\left(1/v'\left(w_1\right)\right)\left(1-\delta\right)\left(q'\right)\left(v\left(b\right)-v\left(w_{2,I}\right)\right) = 0$$

where the last term on the left hand side is the extra cost of compensating period-1 hires for their increased likelihood of replacement (defining $\tilde{q}'$ as before). Differentiating with respect to $\overline{n}_2$,

$$f'\left(n;x\right)q = w_{2,N}q + k. \tag{B.10}$$

Thus, employment is on the labour demand curve, as in (8). We can combine these latter two equations to obtain

$$\left(k/q\right)\tilde{q}'\overline{n}_2 + (1-\delta)n_1\tilde{q}'\left(\left(w_{2,I}-w_{2,N}\right)+\left(1/v'\left(w_1\right)\right)\left(v\left(b\right)-v\left(w_{2,I}\right)\right)\right) = \\ q\left((1-\delta)n_1+\overline{n}_2\right)$$

or

$$k\tilde{q}'/q^2 = 1 + (1-\delta)n_1\tilde{q}'\left(\left(w_{2,N}-w_{2,I}\right)+\left(1/v'\left(w_1\right)\right)\left(v\left(w_{2,I}\right)-v\left(b\right)\right)\right)/q\overline{n}_2 + \\ (1-\delta)n_1/\overline{n}_2 \tag{B.11}$$

Both the second and third terms on the right hand side (henceforth RHS) of (B.11) are positive, the second as v is concave, $w_{2,I} = w_1$ from the above, $w_{2,I} > w_{2,N}$ (as replacement occurs) and $b \leq w_{2,N}$. Recall from the proof of Proposition 1 that $\tilde{q}'/q^2$ is decreasing in θ and $w_{2,N}$.Thus, in comparison to the CTR quasi-supply given by (9), at fixed θ, or equivalently fixed n_2 given $n_2 = p\left(\theta_2\right)S_2$ as in Figure 2, $w_{2,N}$ must be lower to satisfy (B.11). Thus, the intersection with the downward sloping (8) must occur at a lower wage and higher employment than in the CTR solution.

Finally, $w_{2,N}^{CTR}\left(x;w_1,n_1\right) < w_1$ because otherwise, the commitment solution could be implemented, which would be superior. ■

B.3 Proof of Proposition 3

Proof. (i) In the CTR model, consider an equilibrium in the absence of asymmetric information. We have that $w_{2,I}$ is independent of the period-2 state x, and $w_{2,N}$ is chosen independently of $w_{2,I}$ to minimize the cost of hiring a new worker in state x. With asymmetric information, the firm has no incentive to misreport since the wage paid to non-separated period 1 hires is constant, while any different $w_{2,N}$ can only increase new-hire costs. The result follows.

(ii) Let $x' := \hat{x} - \varepsilon$, $x'' := \hat{x} + \varepsilon$. Consider an arbitrary sequence $\{\varepsilon_s\}_{s=0,1,\dots}$, $\varepsilon_s > 0$, $\varepsilon_s \to 0$; we show that there is some $\bar{s}$ such that for $s \geq \bar{s}$, wages are equal in both states: $w_{2,I}(x') = w_{2,I}(x'') = w_{2,N}(x') = w_{2,N}(x'')$.[64] By the assumptions of continuity and the binding no-undercutting condition at $\hat{x}$,

$$\lim_{s\to\infty} w_{2,I}(x') = \lim_{s\to\infty} w_{2,I}(x'') = \lim_{s\to\infty} w_{2,N}(x') = \lim_{s\to\infty} w_{2,N}(x'') = \hat{w}_{2,2} = \hat{w}_{2,1}, \tag{B.12}$$

where the original equilibrium corresponding to $\hat{x}$ is denoted by $\hat{}$. In what follows, we will deal with the case where $w_{2,I}(x') \leq w_{2,I}(x'')$ infinitely often as $s = 0, 1, \dots$, so we consider below the circumstances in which this is true; the arguments apply equally to the opposite case. To consider this case, we define

$$C(w_{2,N}, x'') := (k/q(\theta_2(w_{2,N}, Z_2(x''))) + w_{2,N})$$

and

$$w^{**}(x'') \in \arg\min_{w_{2,N}} (k/q(\theta_2(w_{2,N}, Z_2(x''))) + w_{2,N}) \tag{B.13}$$

where $\theta_2(w_{2,N}, Z_2(x''))$ is as defined in (3); $C(w_{2,N}, x'')$ is the cost per period-2 hire in state x'' ($k/q + w$ is the total cost of a new hire), while $w^{**}(x'')$ is the wage that minimizes this cost. It is independent of the number of hires, and the cost is strictly convex in $w_{2,N}$ (hence, $w^{**}(x'')$ is unique).

To see this, as earlier, write $q(\theta_2(w_{2,N}, Z_2(x''))) \equiv \tilde{q}(w_{2,N}, x'')$, so

[64] The dependence of values on ε_s will mostly be left implicit to avoid the notation becoming more cluttered.

$$\frac{dC\left(w_{2,N},x''\right)}{dw_{2,N}} = -\frac{k\tilde{q}'}{\tilde{q}^2}+1 \tag{B.14}$$

$$= -\frac{k}{\tilde{q}^2}\frac{\theta_2\frac{dq}{d\theta_2}}{\left(1-\frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)}\frac{v'\left(w_{2,N}\right)}{v\left(w_{2,N}\right)-v\left(b\right)}+1,$$

using (B.7). Given that $\tilde{q}' > 0$ (a higher wage increases the job-filling rate), the second term in the product is $q\left(\theta_2\right)\epsilon_q\left(\theta_2\right)/\left(1-\epsilon_q\left(\theta_2\right)\right)$ and therefore is decreasing in θ_2 (by assumption) and, hence, also decreasing in $w_{2,N}$, while the final term in the product is also decreasing in $w_{2,N}$, we have

$$\frac{d^2C\left(w_{2,N},x''\right)}{dw_{2,N}^2} > 0. \tag{B.15}$$

Additionally, given the assumption that the no-undercutting condition is strictly binding initially, we have $\hat{w}_{2,1} > w^{**} := w^{**}\left(x\right)$ (the value for $w^{**}\left(x''\right)$ when $\varepsilon = 0$, being equal to the optimal hiring wage in the unperturbed model), and therefore, by (B.12) and the continuity of $w^{**}\left(x'\right)$ and $w^{**}\left(x''\right)$ in ε (by the Theorem of the Maximum, as they are both unique by the strict convexity of C and C is continuous in Z and, hence, in ε),

$$\lim_{s\to\infty} w^{**}\left(x'\right) = \lim_{s\to\infty} w^{**}\left(x''\right) = w^{**} < \hat{w}_{2,1}. \tag{B.16}$$

Profits in period 2, in state x'', are

$$\max_{n_2}\left(f\left((1-\delta)n_1+n_2;x''\right)-w_{2,I}(x'')(1-\delta)n_1-C\left(w_{2,N}(x''),x''\right)n_2\right).$$

In state x'', the firm can claim that x' occurred and make nonnegative savings in wages paid to incumbents because $w_{2,I}\left(x'\right)\leq w_{2,I}\left(x''\right)$. It follows that we must have

$$C\left(w_{2,N}\left(x''\right),x''\right)\leq C\left(w_{2,N}\left(x'\right),x''\right) \tag{B.17}$$

since otherwise, by announcing x', hiring costs are reduced as well.

There are three possibilities to consider, and at least one of which must occur infinitely often along the sequence $s = 0,1,\ldots$. First, $w_{2,N}\left(x'\right) < w_{2,N}\left(x''\right)$. From (B.17), $w_{2,N}\left(x'\right) < w^{**}\left(x''\right)$ by (B.15). But as $s\to\infty$, a contradiction occurs in view of $\lim_{s\to\infty} w_{2,N}\left(x'\right) = \hat{w}_{2,1}$ and (B.16).

On the other hand, if $w_{2,N}(x') > w_{2,N}(x'')$, then by (B.17) and (B.15), $w_{2,N}(x') > w^{**}(x'')$. However, we have

$$w_{2,N}(x') > w_{2,N}(x'') \geq w_{2,I}(x'') \geq w_{2,I}(x'),$$

where the second inequality follows from no undercutting and the final inequality by hypothesis. However, consider a change where $w_{2,N}(x')$ is cut to $w_{2,N}(x'')$ and $w_{2,I}(x')$ is increased to $w_{2,I}(x'')$ if it is initially below this value. This changed contract satisfies no undercutting and (trivially) incentive compatibility. The decrease in $w_{2,N}(x')$ reduces hiring costs by (B.12) and (B.16), which imply $w_{2,N}(x') > w^{**}(x')$ for a large s. Additionally, for s large enough, $w_{2,I}(x'') < w_1(\varepsilon_s)$ by the binding no-undercutting condition in Problem A (from Proposition 1, this implies $\hat{w}_{22} < \hat{w}_1$), (B.12) and, by assumption, $\lim_{s\to\infty} w_1(\varepsilon_s) = \hat{w}_1$ using an obvious notation. Then, $v'' < 0$ implies that a small reduction in w_1 to leave V_1 constant will reduce expected wages while leaving hiring constant. Therefore, for a large enough s, the contract is not optimal, contrary to the assumption. The final possibility has $w_{2,N}(x') = w_{2,N}(x'')$. By no undercutting, then,

$$w_{2,N}(x') = w_{2,N}(x'') > w_{2,I}(x'') = w_{2,I}(x'),$$

where the final equality follows by incentive compatibility (otherwise, x' would be announced because incumbent wages would be lower), and the inequality is strict by the assumption that it not a constant wage contract. Similar to the previous case, both $w_{2,I}(x'')$ and $w_{2,I}(x')$ can be increased by the same small amount without violating incentive compatibility or no undercutting, which is compensated by a small reduction in $w_1(\varepsilon_s)$, reducing expected wages paid to period-1 hires. Thus, again, the equilibrium contract is not optimal, contrary to assumption.

(iii) Period-2 profits from the contract for state x in state x' can be written as

$$\pi(x, x') := \max_{n_2}\{f((1-\delta)n_1 + n_2; x') - w_{2,I}(x)(1-\delta)n_1 - C(w_{2,N}(x), x')\, n_2\}.$$

We proceed in a number of steps. (a) Suppose that there is a binding incentive compatibility constraint between states x' and x'' such that $\pi(x', x') = \pi(x'', x')$ and $C(w_{2,N}(x'), x') > C(w_{2,N}(x''), x')$, so the firm benefits from announcing x'' in state x' from the point of view of new-hire costs. Incentive compatibility implies $w_{2,I}(x') < w_{2,I}(x'')$. Then, consider replacing the x' contract by that at x'' (holding n_1 constant). This must trivially satisfy incentive compatibility and no undercutting and leave ex post profits unchanged. However, since $w_{2,I}$ is increased in state x', ex ante utility V_1 rises, which reduces period-1 hiring costs; hence, profits increase, contrary to optimality. We conclude that $\pi(x', x') = \pi(x'', x')$ implies $C(w_{2,N}(x'), x') \leq C(w_{2,N}(x''), x')$, and hence, by incentive compatibility, $w_{2,I}(x') \geq w_{2,I}(x'')$ (and if the first inequality is strict or an equality, so is the second, and vice versa).

(b) Let $X' \subseteq X$ be such that for $x \in X'$, $w_{2,I}(x) > w_1$. We show that $X' = \varnothing$. For $x' \in X'' := X \backslash X'$, $x \in X'$, we cannot have $\pi(x', x') = \pi(x, x')$, since $w_{2,I}(x') < w_{2,I}(x)$, contradicting (a). Hence, $\pi(x', x') > \pi(x, x')$ (incentive compatibility is slack). Hence, we can find (by X finite) an $\eta > 0$ such that $\pi(x', x') \geq \pi(x, x') + \eta$ for all $x' \in X''$, $x \in X'$. Next, cut $w_{2,I}(x)$ by $\varepsilon < \eta((1-\delta)n_1)^{-1}$ for all $x \in X'$; this does not affect incentive compatibility between $x, x'' \in X'$ as profits change by the same amount in each state, and by construction of ε, $\pi(x', x') > \pi(x, x')$, $x' \in X''$, $x \in X'$. As $\pi(x, x)$ is increased for each $x \in X'$ by $\varepsilon(1-\delta)n_1$, $\pi(x, x) > \pi(x', x)$, $x' \in X''$, as the RHS is unchanged and a weak inequality held before the change. Thus, (global) IC is satisfied. No undercutting is satisfied because only $w_{2,I}$ is cut. If $X' \neq \emptyset$, for a small enough ε, this uniform cut in $w_{2,I}$ in all states where $w_{2,I} > w_1$ and a corresponding increase in w_1 to leave V_1 unchanged increases profits by standard consumption smoothing arguments (hold n_1 constant), i.e., a profitable deviation that is contrary to the assumption. We conclude that $X' = \emptyset$, i.e., $w_{2,I}(x') \leq w_1$ all $x' \in X$.

(c) Let $\hat{X} := \arg\max_{\hat{x}} w_{2,I}(\hat{x})$. If this is a singleton, $\{x\}$, then by part (a), there is no other state x' with $\pi(x', x') = \pi(x, x')$. It follows that provided that the no undercutting constraint is slack in state x, $w_{2,N}(x) = w^{**}(x)$ and, hence, $w_{2,N}(x) = w_{2,N}^{CTR}(x, w_1, n_1)$, as otherwise if $w_{2,N}(x) \neq w^{**}(x)$ a small enough change in $w_{2,N}$ towards w^{**} increases profits in state x (by the strict convexity of $C(\cdot, x)$), satisfies no undercutting, violates no $\pi(x', x') \geq \pi(x, x')$ constraint for all $x' \neq x$, and relaxes $\pi(x, x) \geq \pi(x', x)$ for $x' \neq x$. If no undercutting binds in state x, this argument implies $w_{2,N}(x) \geq w^{**}(x)$, as $w_{2,N}$ can be increased if $w_{2,N} < w^{**}$ and, hence, $w_{2,N}(x) \geq w_{2,N}^{CTR}(x, w_1, n_1)$.

If $\hat{X}$ is not a singleton, by a similar argument, consider $x \in \hat{X}$ such that $w_{2,N}(x) \neq w^{**}(x)$. If no undercutting is not binding at state x, change $w_{2,N}(x)$ towards $w^{**}(x)$ by an amount ε such that $C(w_{2,N}(x), x)$ falls. Again, by part

(a) for all $x' \notin \hat{X}$, we have $\pi(x', x') > \pi(x, x')$, and provided that ε is small enough, these incentive compatibility and no undercutting constraints are not violated. If any incentive compatibility constraint for $x'' \in \hat{X}$ is violated, replace $w_{2,N}(x'')$ by the new value of $w_{2,N}(x)$; this increases ex post profits in x'' and does not affect period 1, as $w_{2,I}$ is unchanged. Profits are increased by this change, contrary to the assumption. Hence, $w_{2,N}(x) = w^{**}(x)$ for all $x \in \hat{X}$. If no undercutting binds at the lowest $w_{2,N}(x)$, $x \in \hat{X}$, again, $w_{2,N}(x) \geq w^{**}(x)$.

(iv) Follows from (iii) (b) above. ■

B.4 Proof of Proposition A.1

Proof. Incentive compatibility in state x'' requires that

$$\begin{aligned}(f((1-\delta)n_1 + n_2(x''); x'') - w_{2,I}(x'')(1-\delta)n_1 - C(w_{2,N}(x''), x'')n_2(x'')) \geq \\ (f((1-\delta)n_1 + n_2(x'); x'') - w_{2,I}(x')(1-\delta)n_1 - C(w_{2,N}(x'), x'')n_2(x')),\end{aligned} \tag{B.18}$$

where $C(\cdot,\cdot)$ is the total cost of a new period 2 hire as defined as in the proof of Proposition 3, and hiring in state x' is denoted $n_2(x')$, etc. We will write $w_{2,N}(x')$ as $w'_{2,N}$ etc. to simplify notation below.

We start by assuming that the optimal contract is differentiable (from the right) at $\varepsilon = 0$. Consider ε small and take a first-order approximation for (B.18) around the initial equilibrium[65] at $\hat{x}$, where (B.18) trivially holds with equality (and where as in the proof of Proposition 3 we use a ˆ to denote the corresponding initial equilibrium contract) and defining deviations as $\Delta w'_{2,I} := w'_{2,I} - \hat{w}_{2,2}$ etc., and where $\Delta x''(= -\Delta x') := x'' - \hat{x} = \varepsilon$: $f'(\Delta n''_2 - \Delta n'_2) - (1-\delta)n_1(\Delta w''_{2,I} - \Delta w'_{2,I}) - \frac{\partial C}{\partial w} n_2 (\Delta w''_{2,N} - \Delta w'_{2,N}) - C(\Delta n''_2 - \Delta n'_2) \geq 0$, with the reverse inequality implied by incentive compatibility in state x', so given that $f' = C$ in the initial equilibrium ($\hat{n}_2$ is chosen efficiently given $\hat{w}_{2,1}$ in the absence of incentive compatibility constraints), we get

$$-(1-\delta)n_1(\Delta w''_{2,I} - \Delta w'_{2,I}) - \frac{\partial C}{\partial w} n_2 (\Delta w''_{2,N} - \Delta w'_{2,N}) = 0. \tag{B.19}$$

Suppose that $\Delta w''_{2,I} < \Delta w'_{2,I}$; we will establish a contradiction. Since $\frac{\partial C}{\partial w} > 0$

[65] That is, we omit terms of order smaller than ε in the expressions that follow. We assumed that the equilibrium of the model is differentiable in ε on an interval $[0, \bar{\varepsilon})$ (from the right at 0), so that in particular C is also differentiable in x. In the approximation $\partial C/\partial x$ cancels.

(at the initial equilibrium), (B.19) implies $sgn\left(\Delta w''_{2,I} - \Delta w'_{2,I}\right) = - sgn\left(\Delta w''_{2,N} - \Delta w'_{2,N}\right)$. Hence $\Delta w''_{2,N} > \Delta w'_{2,N}$; thus $w''_{2,I} < w'_{2,I}$ and $w''_{2,N} > w'_{2,N}$ and

$$w''_{2,I} < w'_{2,I} \leq w'_{2,N} < w''_{2,N},$$

where the weak inequality follows by no undercutting in state x'.

Consider the following change to the contract (use a $\tilde{}$ to denote this new contract): set wages in x'' to equal those in x': increase $w''_{2,I}$ to $\tilde{w}''_{2,2} := w'_{2,I}$ and reduce $w''_{2,N}$ to $\tilde{w}''_{2,1} = w'_{2,N}$; hold n_1 constant, set n_2 in each state to maximize period 2 profits given $w'_{2,N}$ and $\tilde{w}''_{2,1}$, and change w_1 to $\tilde{w}_1$ to keep V_1 constant. The cut in $w''_{2,N}$ reduces hiring costs by, for ε small enough, $w''_{2,N} > w^{**}(x'')$ (the latter being the new-hire cost minimizing wage in state x'', using notation and the argument in the proof of Proposition 3 above) and as $\tilde{n}''_2$ is chosen optimally, profits on new hires in x'' must rise. Likewise as $\tilde{n}'_2$ is chosen optimally profits in x' cannot fall. Incentive compatibility is satisfied trivially. From V_1 constant (which implies constant job opening creation and hence constant period 1 job opening costs),

$$v(\tilde{w}_1) - v(w_1) + 0.5\beta(1-\delta)\left(v\left(w'_{2,I}\right) - v\left(w''_{2,I}\right)\right) = 0. \tag{B.20}$$

By $w^*_{2,I} < w^*_1$, $w'_{2,I} < w_1$; also $w'_{2,I} < \tilde{w}_1$ for ε small enough, so

$$w_1 > \tilde{w}_1 > w'_{2,I} > w''_{2,I}.$$

It follows from (B.20) and by $v'' < 0$ that

$$w_1 - \tilde{w}_1 > 0.5(1-\delta)\left(w'_{2,I} - w''_{2,I}\right);$$

thus the change in costs of period 1 hires is

$$n_1\left(\tilde{w}_1 - w_1 + 0.5(1-\delta)\left(w'_{2,I} - w''_{2,I}\right)\right) < 0.$$

Thus the new contract is more profitable than the putative equilibrium one, a contradiction. This establishes that $\Delta w''_{2,I} < \Delta w'_{2,I}$ is not possible. Similarly $\Delta w''_{2,I} > \Delta w'_{2,I}$ yields a contradiction. Thus $\Delta w''_{2,I} = \Delta w'_{2,I}$ and so by (B.19) $\Delta w''_{2,N} = \Delta w'_{2,N}$. It follows that $\left(\Delta w''_{2,N} - \Delta w'_{2,N}\right)/(2\varepsilon) = 0$, which establishes the claim.

Now we allow for the contract to be non-differentiable in ε (from the right) at $\varepsilon = 0$. It must be (right) continuous at $\varepsilon = 0$, as otherwise profits would also be discontinuous, while a simple constant wage contract would be continuous so would do better.[66] Consider a sequence for $\varepsilon \equiv (x'' - x')/2$: $\{\varepsilon_\nu\}$, $\varepsilon_\nu \to 0$ as $\nu \to \infty$. Assume that the no undercutting constraint binds (so that $w_{2,I} = w_{2,N} =: w_2$ say) in both states along the sequence (cf. proof of Proposition 3) and that only the downward incentive constraint binds (i.e., (B.18)). Then by standard arguments $w_2'' \geq w_2'$ and n_2'' is at the optimal level given w_2''.[67] The other possibilities can be dealt with in an analogous manner. We again suppress the explicit dependence of the optimal contract on ε_ν for notational simplicity. We suppose, contrary to hypothesis, that

$$0 < \limsup_{\nu \to \infty} |w_2'' - w_2'| / \varepsilon_\nu. \tag{B.21}$$

Rearranging (B.18):

$$f\left((1-\delta)n_1 + n_2''; x''\right) - f\left((1-\delta)n_1 + n_2'; x''\right) - C\left(w_{2,N}\left(x''\right), x''\right) n_2\left(x''\right) + C\left(w_{2,N}\left(x'\right), x''\right) n_2\left(x'\right) - (1-\delta)n_1\left(w_2'' - w_2'\right) \geq 0. \tag{B.22}$$

By (B.21) we can take a subsequence such that $\lim_{\nu \to \infty} \left(w_2'' - w_2'\right)/\varepsilon = a$ where $|a| > 0$, and where n_1 converges to say $\tilde{n}_1$, we get after dividing (B.22) by ε_ν and taking the limit:

$$\liminf_{\nu \to \infty} [(f\left((1-\delta)n_1 + n_2''; x''\right) - f\left((1-\delta)n_1 + n_2'; x''\right) - C\left(w_2'', x''\right) n_2'' + C\left(w_2', x''\right) n_2')/\varepsilon_\nu] \geq (1-\delta)\tilde{n}_1 a. \tag{B.23}$$

By $w_2'' - w_2' \geq 0$, $a > 0$. In other words, assuming for small ε we have lower wages in state x' than in x'' by a first-order amount, implies that the RHS of (B.23) is positive, that is, there is a (first-order) incentive in state x'' to underreport x to benefit from lower wage costs; to offset this (i.e., to preserve

[66] Profits are bounded above by a contract which ignores the incentive constraint, which would be continuous, so any discontinuity must imply profits jump down for $\varepsilon > 0$. Holding wages constant across states and setting period 2 employment efficiently at those wages as in the construction in the proof of Proposition 3 would satisfy incentive constraints and lead to profits varying continuously; hence this would be a profitable deviation.

[67] I.e., it maximizes $f\left((1-\delta)n_1 + n_2''; x''\right) - C\left(w_2'', x''\right) n_2''$.

incentive compatibility) the level of new hires in state x' needs to be sufficiently different (below in this case) that in x'' to lead to a fall in profits from new hires that is also first-order. We show that such a difference in hires would also imply, contrary to optimality, that a deviation contract is profitable which avoids the costs of distorted employment, where wages are constant and employment in state x' is set at an efficient level given wages.

Consider then the following possible deviation contract. In state x' set $w_{2,N} = w_{2,I} = w_2''$, and set n_2 at the profit maximizing level in state x' for w_2'', say $\tilde{n}_2'$. Change w_1 to leave V_1 unchanged (and leave hiring in period 1 the same). In period 2 this contract differs only in state x', satisfies no undercutting, and is incentive compatible as wages are the same across states and n_2 is chosen optimally in each state. Considering only incumbents the wage increase from w_2' to w_2'' must increase profits once the reduction in w_1 is taken into account ($w_2' < w_1$ implies that more smoothing reduces wage costs). As overall profits cannot be improved by any deviation, the change in profits in state x' ignoring incumbents must be nonpositive, i.e.,

$$
\begin{aligned}
0 \geq \\
&(f((1-\delta)n_1 + \tilde{n}_2'; x') - C(w_2'', x')\tilde{n}_2') - (f((1-\delta)n_1 + n_2'; x') - C(w_2', x')n_2') \geq \\
&(f((1-\delta)n_1 + n_2''; x') - C(w_2'', x')n_2'') - (f((1-\delta)n_1 + n_2'; x') - C(w_2', x')n_2'),
\end{aligned} \tag{B.24}
$$

where the second inequality follows by definition of $\tilde{n}_2'$ yielding at least as much profit as n_2'' at w_2''. Dividing the RHS of (B.24) by ε_ν, note that this differs from the term in square brackets in (B.23) only by the argument in x, so that given differentiability of f and C in x the two expressions differ by a term of order less than ε_ν.[68] So taking the limit as $\nu \to \infty$, we get the same value, which is a contradiction as from (B.23) it is at least $(1-\delta)\tilde{n}_1 a > 0$, whereas from (B.24) it is nonpositive. ■

[68] I.e., by a term $h(\varepsilon) = o(\varepsilon)$ so that $h(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$. This follows as the derivative of the RHS of (B.24) with respect to x at the limit contract, i.e., the initial ($\varepsilon = 0$) contract, equals zero. Recall that by continuity n_2', n_2'', converge to the same value, etc.

C Further Tables

Table C.1: GDP, CPI, Population, and Unemployment Rate

Year	Nominal GDP (in Mill. Euros)	CPI	Population (in 1,000)	Unemployment rate (in %)
1978	678,940	47.6	61,322	4.3
1979	737,370	49.5	61,439	3.8
1980	788,520	52.2	61,658	3.8
1981	825,790	55.5	61,713	5.5
1982	860,210	58.4	61,546	7.5
1983	898,270	60.3	61,307	9.1
1984	942,000	61.8	61,049	9.1
1985	984,410	63.0	61,020	9.3
1986	1,037,130	63.0	61,140	9
1987	1,065,130	63.1	61,238	8.9
1988	1,123,290	63.9	61,715	8.7
1989	1,200,660	65.7	62,679	7.9
1990	1,306,680	67.5	63,726	7.2
1991	1,415,800	70.2	64,485	6.2
1992	1,485,759	73.8	65,289	6.4
1993	1,503,858	77.1	65,740	8.0
1994	1,556,575	79.1	66,007	9.0
1995	1,606,164	80.5	66,342	9.1
1996	1,625,847	81.6	66,583	9.9
1997	1,664,512	83.2	66,688	10.8
1998	1,711,722	84.0	66,747	10.3
1999	1,751,665	84.5	66,946	9.6
2000	1,799,706	85.7	67,140	8.4
2001	1,856,557	87.4	65,323	8.0
2002	1,879,896	88.6	65,527	8.5
2003	1,888,205	89.6	65,619	9.3
2004	1,933,051	91.0	65,680	9.4
2005	1,960,396	92.5	65,698	11
2006	2,038,803	93.9	65,667	10.2
2007	2,142,032	96.1	65,664	8.3
2008	2,180,829	98.6	65,541	7.2
2009	2,088,073	98.9	65,422	7.8
2010	2,191,138	100.0	65,426	7.4
2011	2,298,449	102.1	64,429	6.7
2012	2,345,295	104.1	64,619	6.6
2013	2,401,853	105.7	64,848	6.7
2014	2,483,514	106.7	65,223	6.7

Note: Identified downswing years are indicated in bold year numbers. Real GDP per capita calculated using nominal GDP, CPI, and population. Sources for the nominal GDP for West Germany: German Federal Statistical Office & the Federal Statistical Offices of the Federal States. Source German CPI: Federal Reserve Bank of St. Louis (FRED Economic Data). Source West German Population: German Federal Statistical Office. Source West German unemployment rate (in % of total civilian workforce): Sachverständigenrat.

Table C.2: Number of Spells of Incumbent and Newly Hired Workers

Year	New Hires	Incumbents	Year	New Hires	Incumbents
1978	536,480	860,131	1997	481,019	2,405,614
1979	580,482	1,070,423	1998	524,318	2,392,430
1980	562,231	1,254,231	1999	580,765	2,385,722
1981	472,966	1,423,195	2000	601,915	2,445,300
1982	383,748	1,535,036	2001	558,655	2,454,149
1983	384,038	1,607,852	2002	471,745	2,444,711
1984	421,761	1,650,744	2003	424,415	2,505,278
1985	433,296	1,703,623	2004	395,014	2,473,805
1986	480,197	1,829,471	2005	391,361	2,443,718
1987	467,208	1,925,379	2006	441,206	2,449,759
1988	501,192	2,008,610	2007	487,477	2,465,401
1989	580,223	2,080,315	2008	474,157	2,506,474
1990	674,453	2,164,259	2009	400,230	2,502,328
1991	651,557	2,284,766	2010	462,299	2,502,616
1992	569,494	2,394,251	2011	444,522	2,409,295
1993	482,607	2,431,712	2012	430,893	2,480,722
1994	496,822	2,428,188	2013	418,203	2,519,325
1995	516,571	2,416,687	2014	432,368	2,521,718
1996	481,872	2,408,716	Total	18,097,160	79,785,954

Note: New hires identified using the first employment spell in an establishment.

Table C.3: Classification of Economic Activities, Edition 1993 (WZ 93)

Section	Description
A	Agriculture, hunting and forestry
B	Fishing
C	Mining and Quarrying
D	Manufacturing
E	Electricity, Gas and water supply
F	Construction
G	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods
H	Hotels and restaurants
I	Transport, storage and communication
J	Financial intermediation
K	Real estate, renting and business activities
L	Public administration and defence; compulsory social security
M	Education
N	Health and social work
O	Other community, social and personal service activities
P	Private households with employed persons
Q	Extra-territorial organizations and bodies

Note: For some analyses we examine the behaviour of wages in each of six broad sectors. The sectors are: Sector 1 (Mining, Agriculture, etc.) includes the WZ 93 sections A to C. Sector 2 (Manufacturing) equals section D, Sector 3 (Power) equals section E, Sector 4 (Construction) equals section F, and Sector 5 (Retail) equals section G. Sector 6 (all other activities) includes sections H to Q.

Source: Federal Statistical Office (Ed.) (2008).

References

Federal Statistical Office (Ed.) (2008). Structure of the classification of economic activities, edition 2008 (WZ 2008). Federal Statistical Office Germany, Wiesbaden.